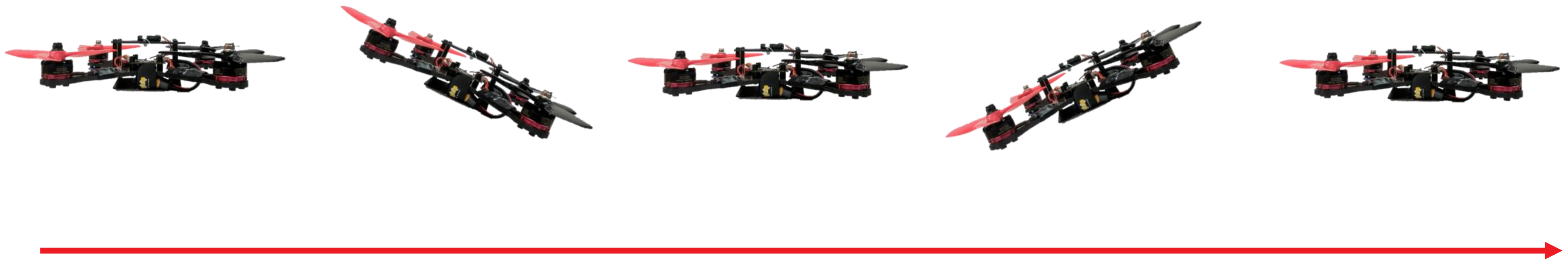
CMSC828T
Vision, Planning And Control In Aerial Robotics

QUADROTOR DYNAMICS

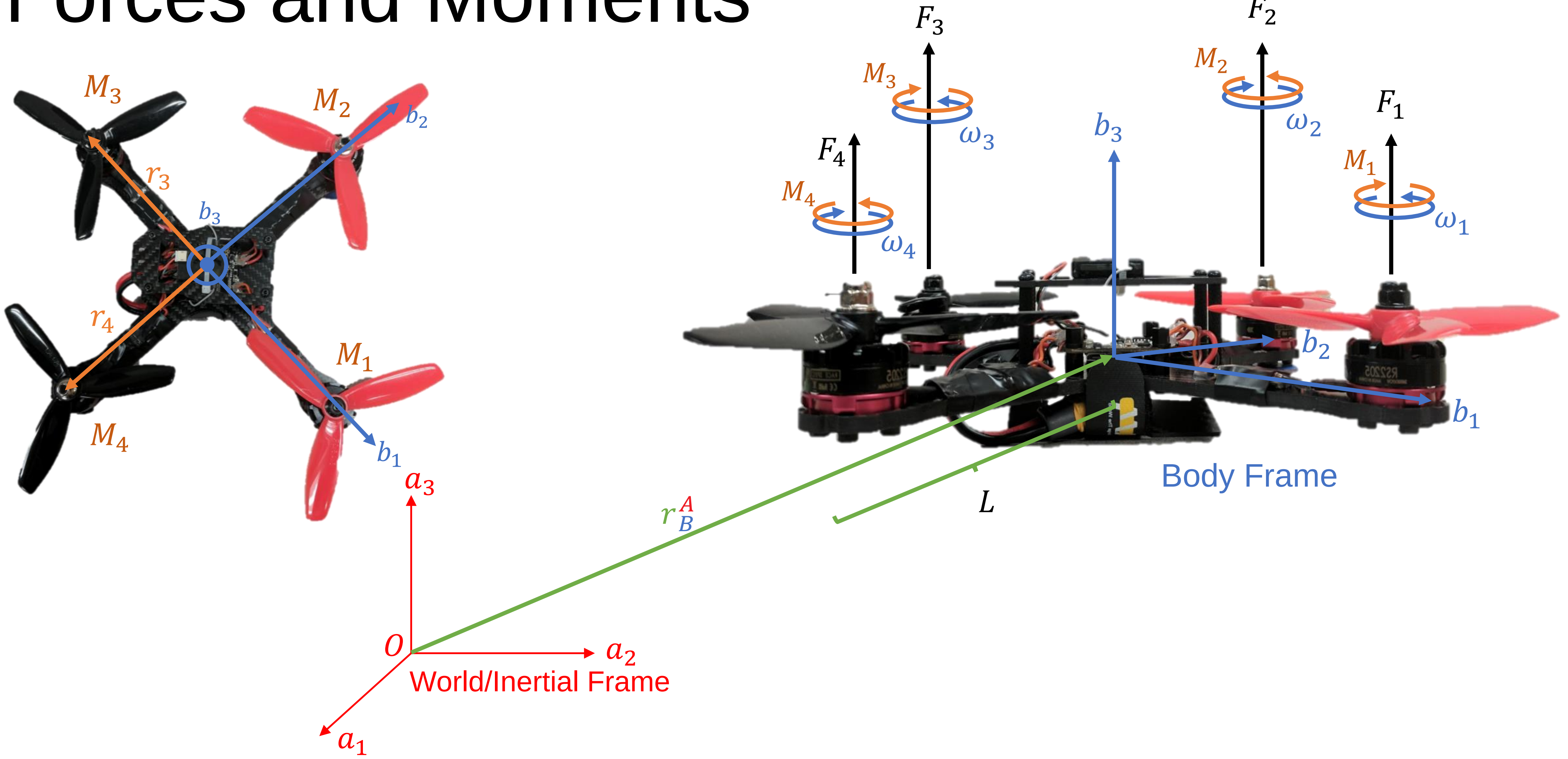
Why is Dynamics Important?

Point A to Point B



Most of these slides are
inspired by MEAM620 Slides at UPenn

Forces and Moments



Forces and Moments

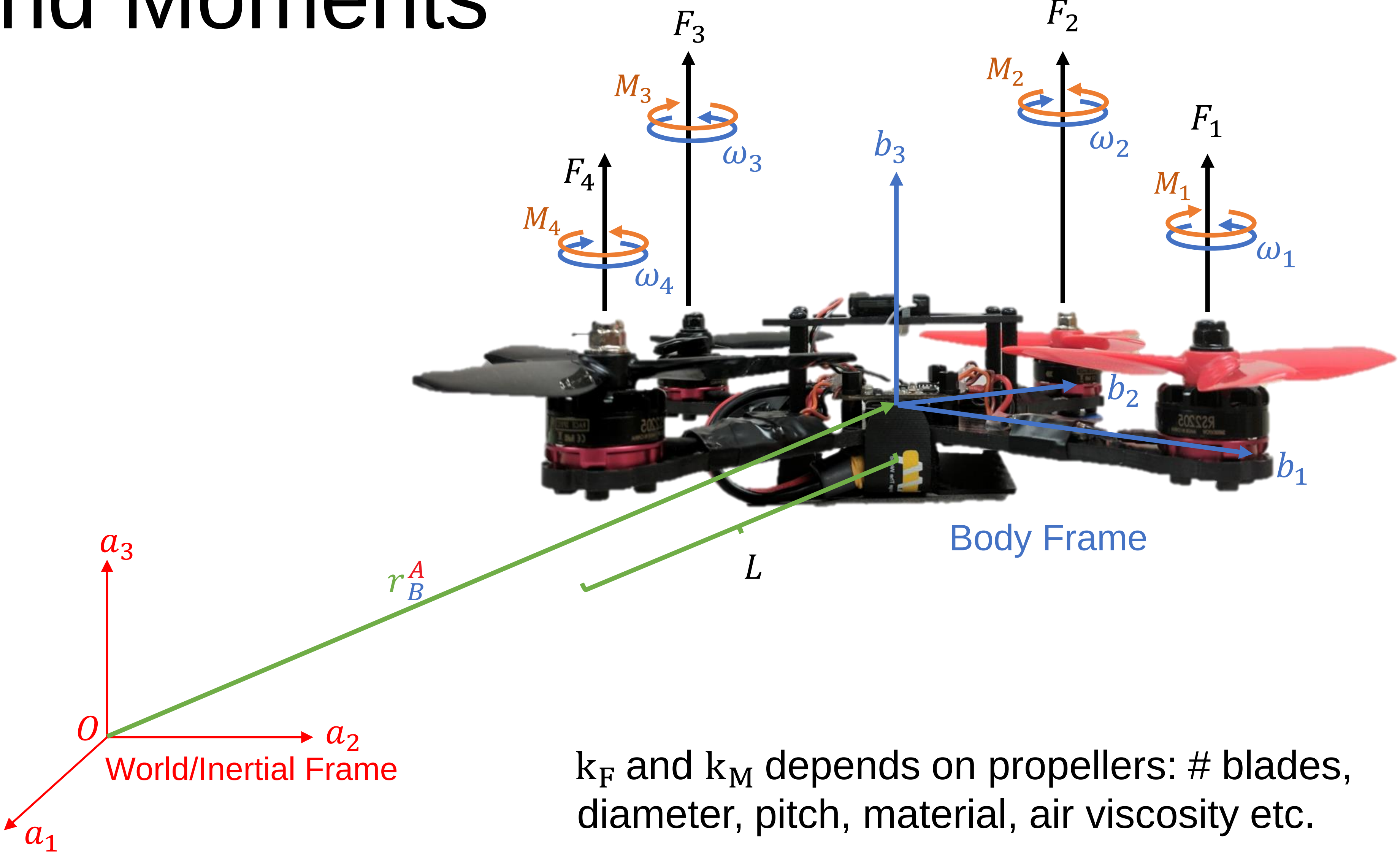
Recall fluid dynamics,

$$F_i \propto \omega_i^2$$

$$F_i = \mathrm{k_F}\omega_i^2$$
$$M_i = \mathrm{k_M}\omega_i^2$$

Net Force:

$$F = \sum F_i - \mathrm{mg}b_3$$
$$i \in \{1,2,3,4\}$$



$\mathrm{k_F}$ and $\mathrm{k_M}$ depends on propellers: # blades, diameter, pitch, material, air viscosity etc.

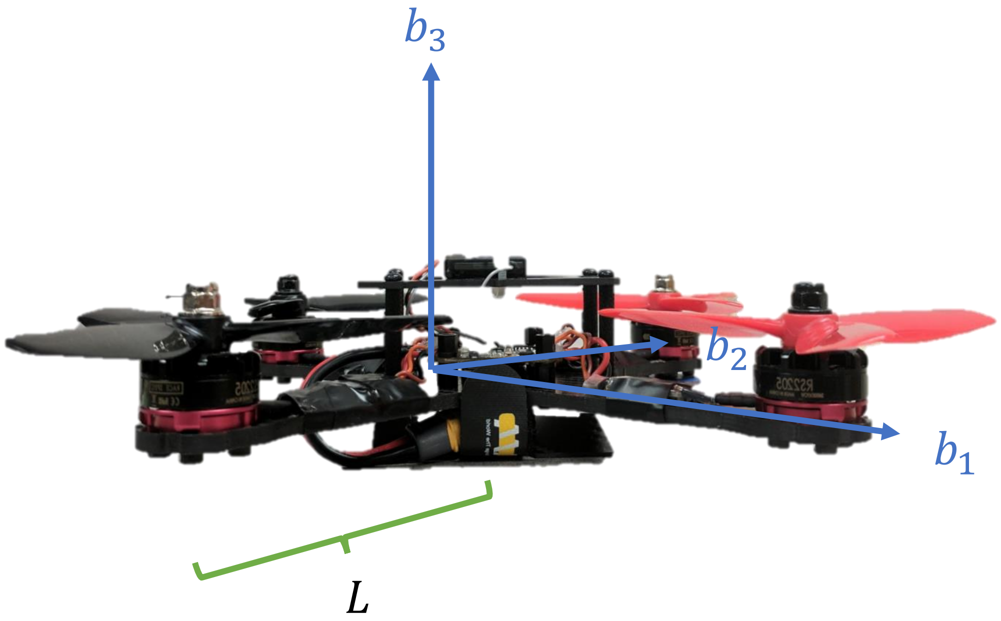
Newton-Euler Equation for a Quadrotor

$^{A}\omega^{B} = p b_1 + q b_2 + r b_3$ Angular velocities in body frame

In Inertial frame:

$$m\ddot{r} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R_B^A \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$$

u_1



Recall, Euler's rotation equation:

$$M = I\dot{\omega} + \omega \times (I\omega)$$

Now, in body frame:

u_2

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

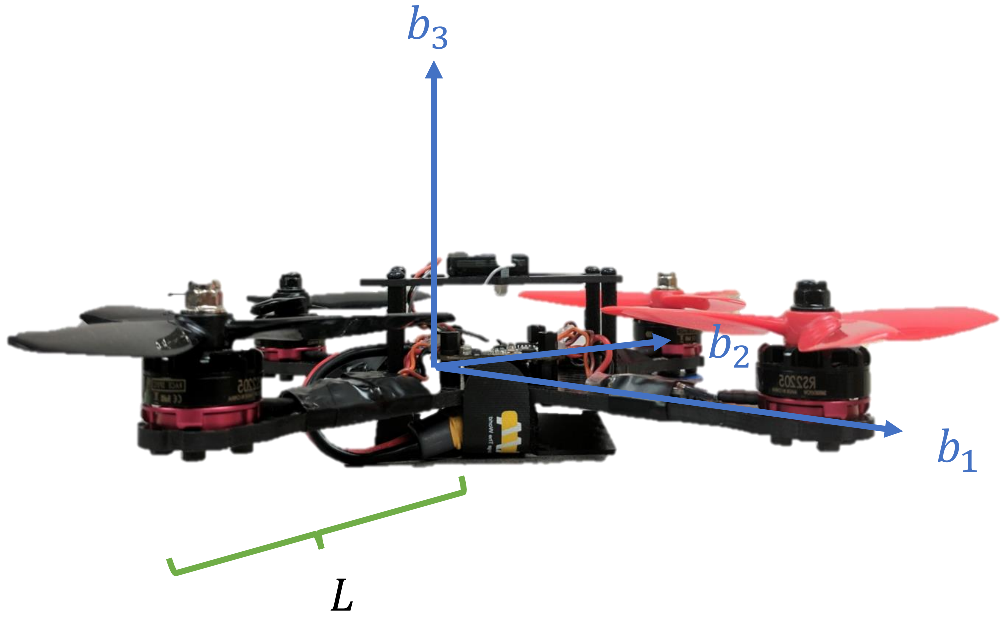
Newton-Euler Equation for a Quadrotor

Remember: $F_i = k_F \omega_i^2$ and $M_i = k_M \omega_i^2$

Let $\gamma = \frac{k_M}{k_F} = \frac{M_i}{F_i}$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & L & 0 & -L \\ -L & 0 & L & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}}_{u_2} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

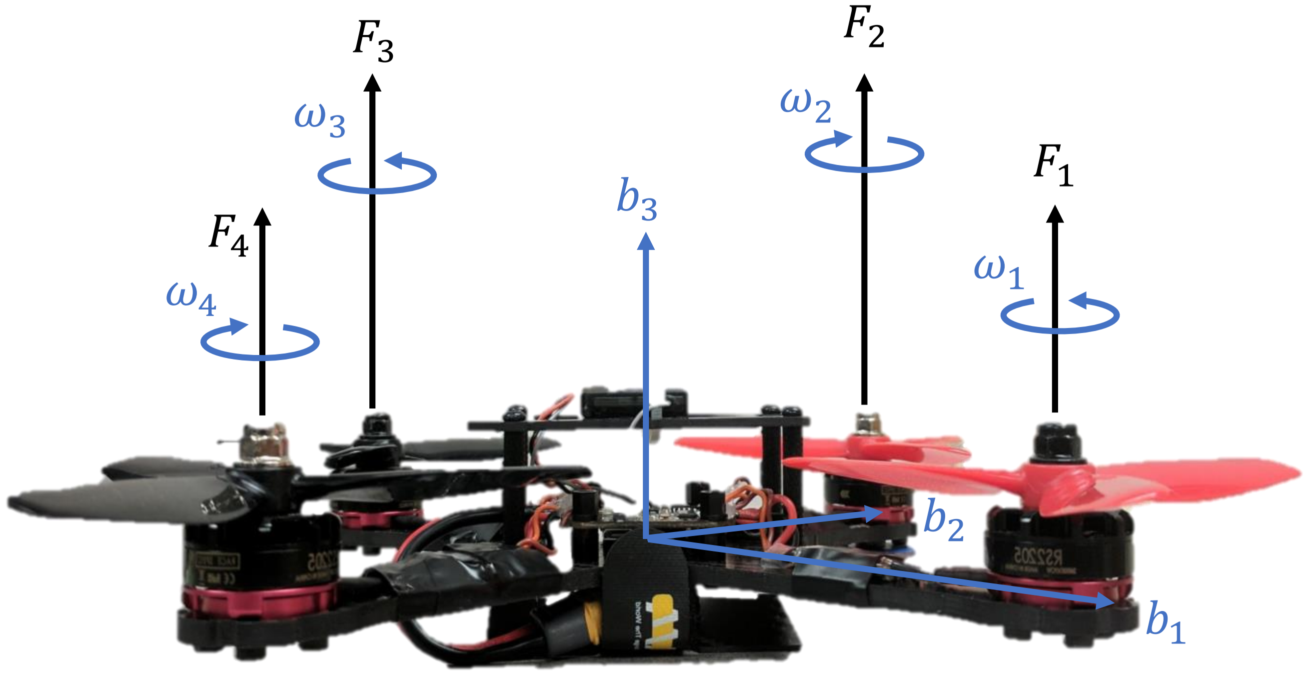


Controller Inputs

$$u = \begin{bmatrix} \color{red}{u_1} \\ \color{green}{u_2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & L & 0 & -L \\ -L & 0 & L & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$$= \begin{bmatrix} \color{red}{\text{thrust}} \\ \color{green}{\text{moment}_x} \\ \color{green}{\text{moment}_y} \\ \color{green}{\text{moment}_z} \end{bmatrix}$$

Everything is in the body frame!



Body Frame