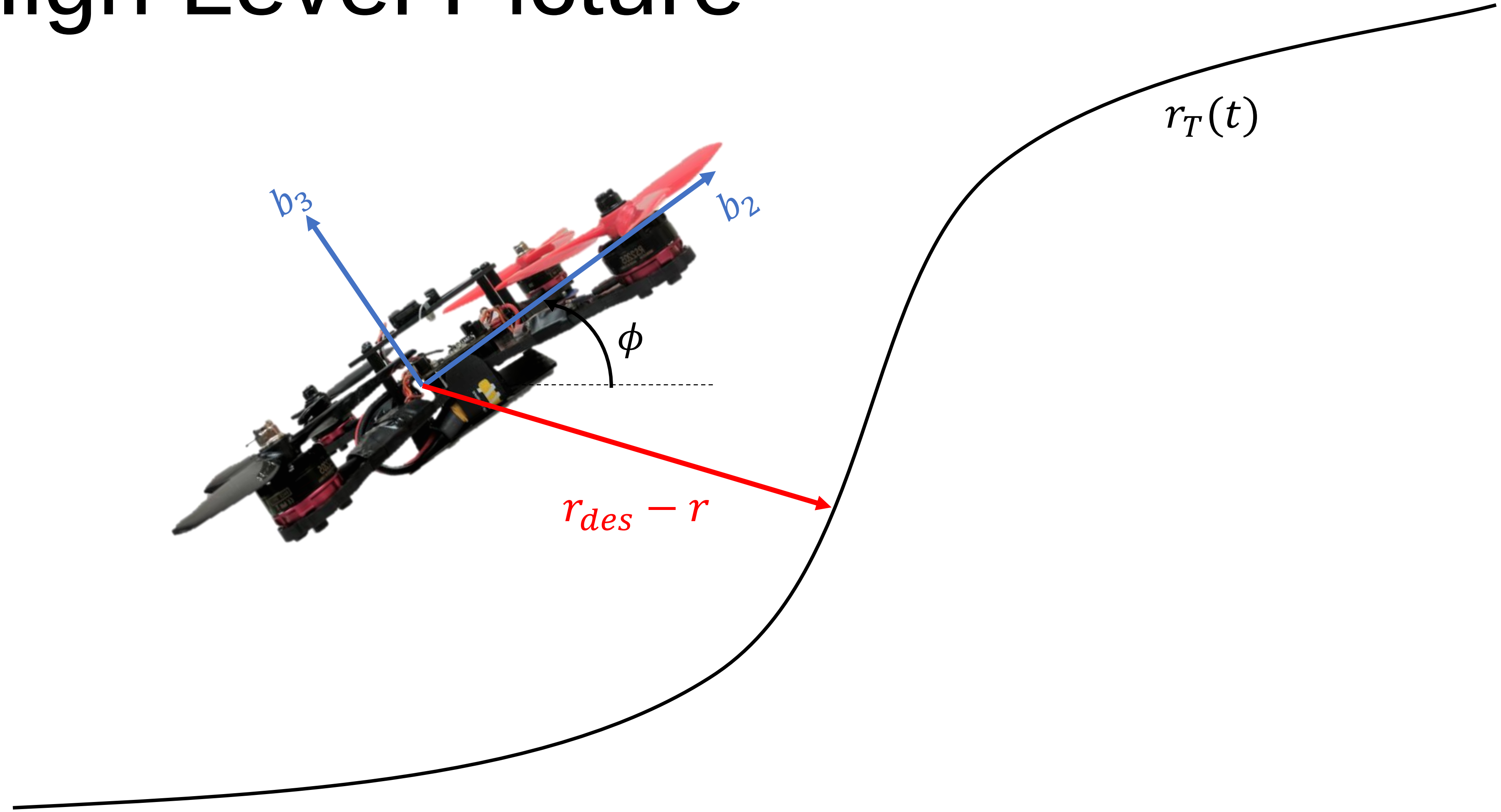
CMSC828T
Vision, Planning And Control In Aerial Robotics

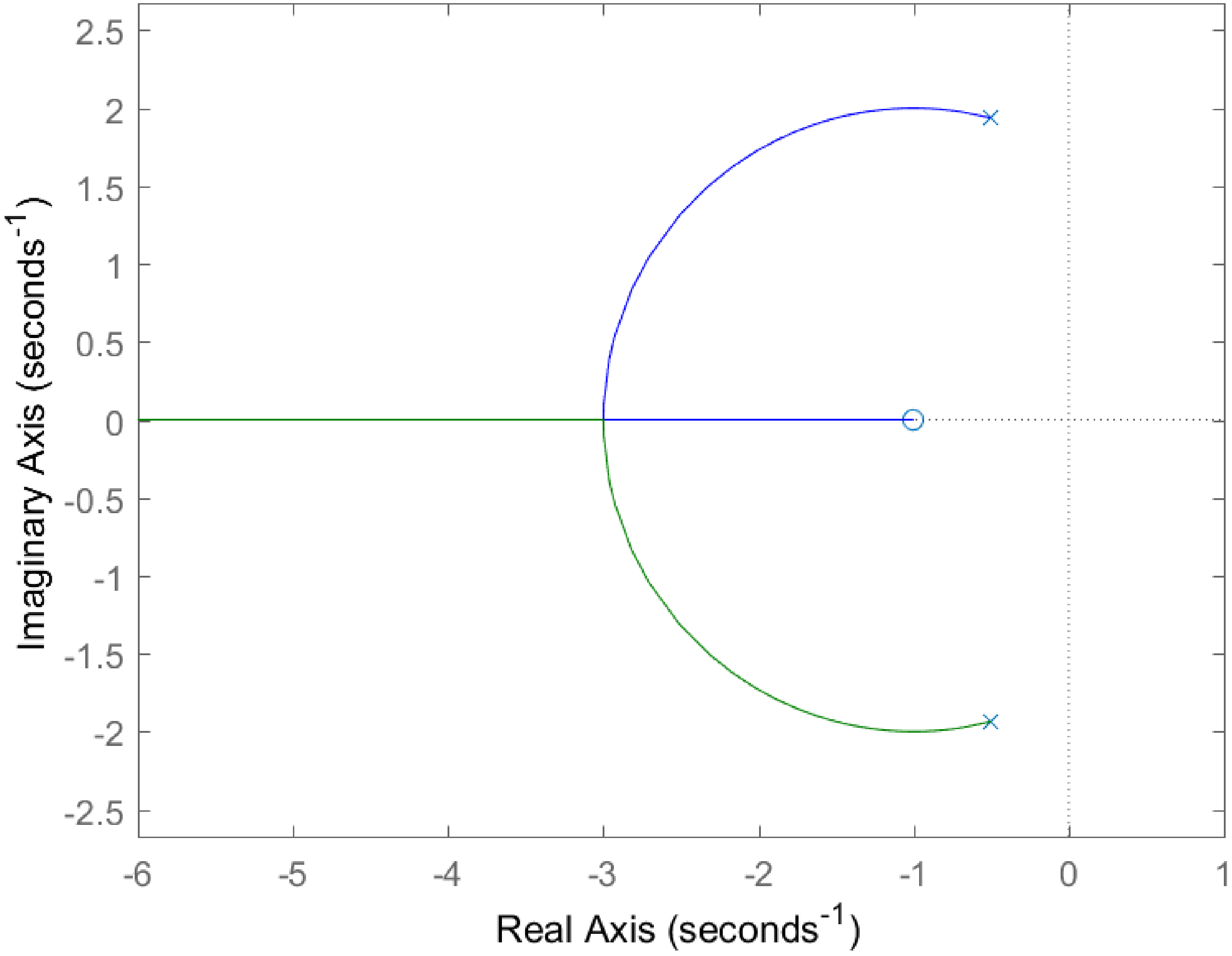
QUADROTOR CONTROL

High Level Picture



Most of these slides are
inspired by MEAM620 Slides at UPenn

Root Locus Plot



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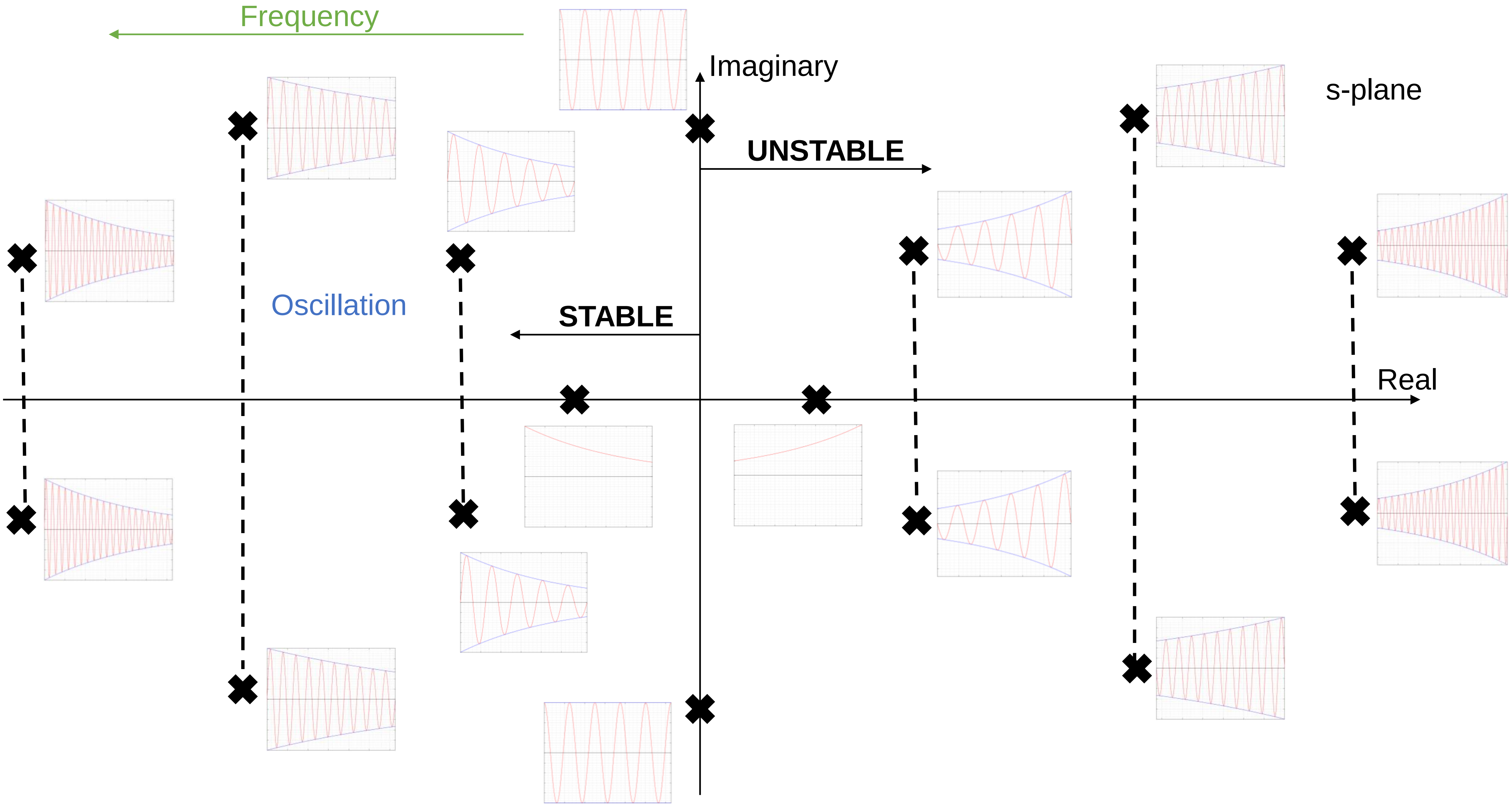
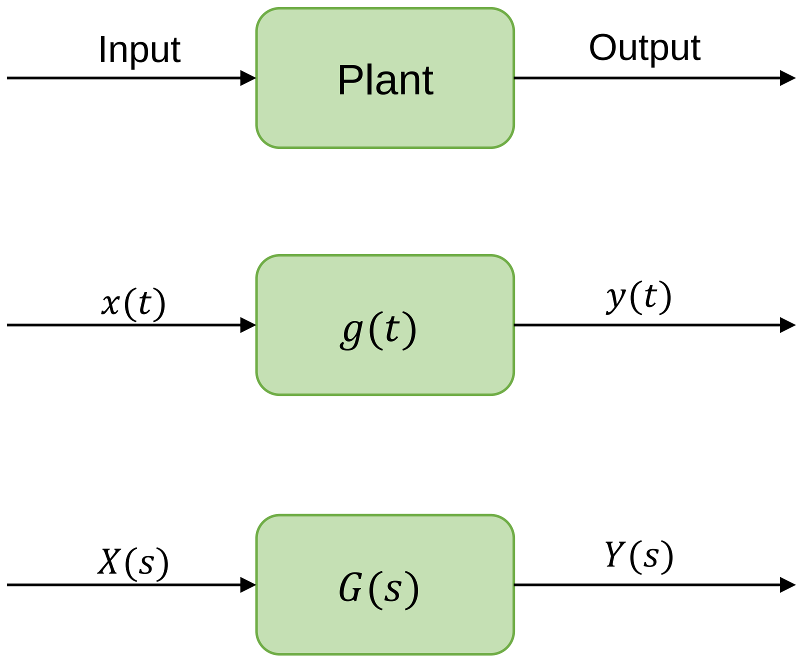
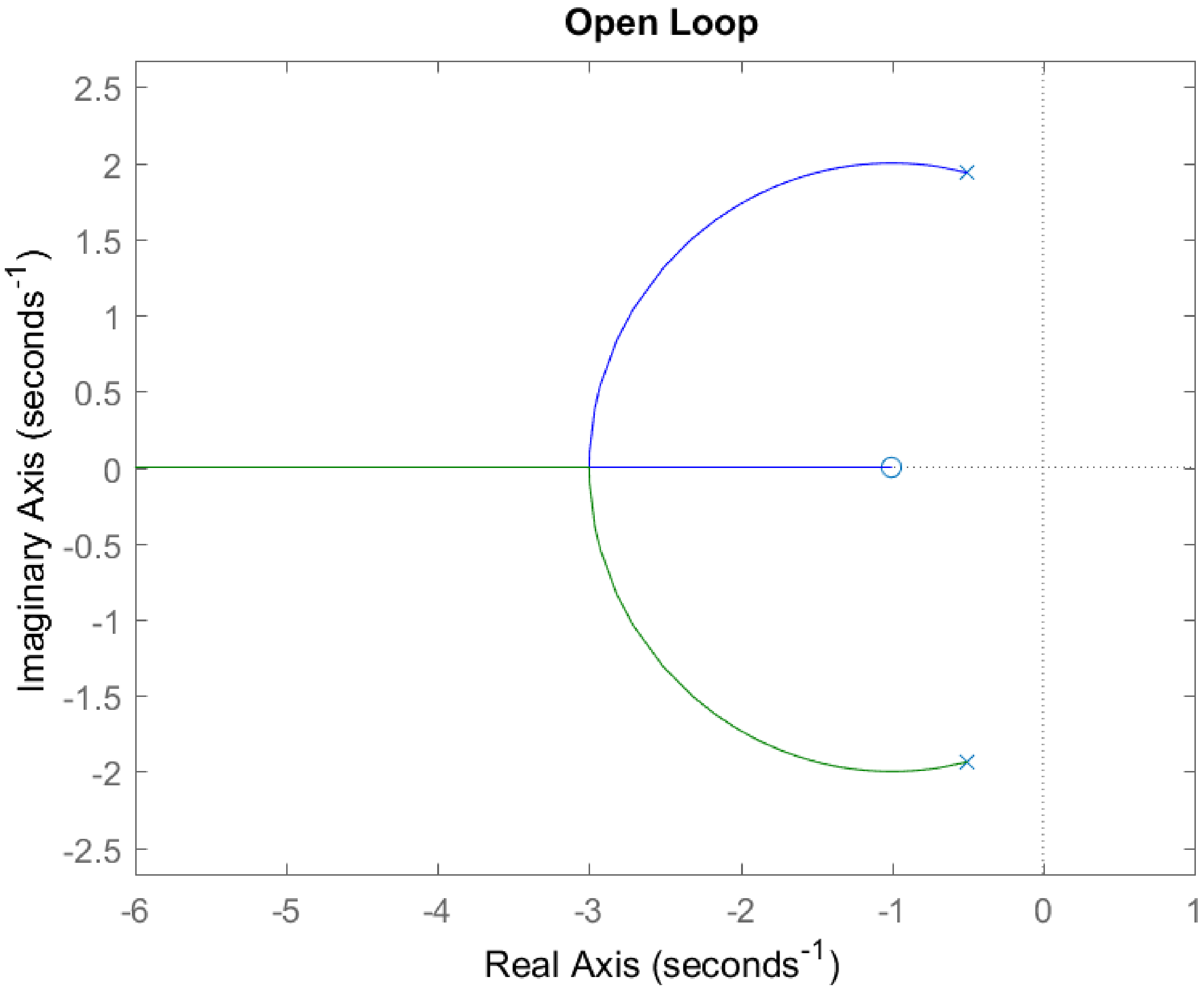

Frequency
Imaginary
s-plane
UNSTABLE
Oscillation
STABLE
Real

Open Loop System



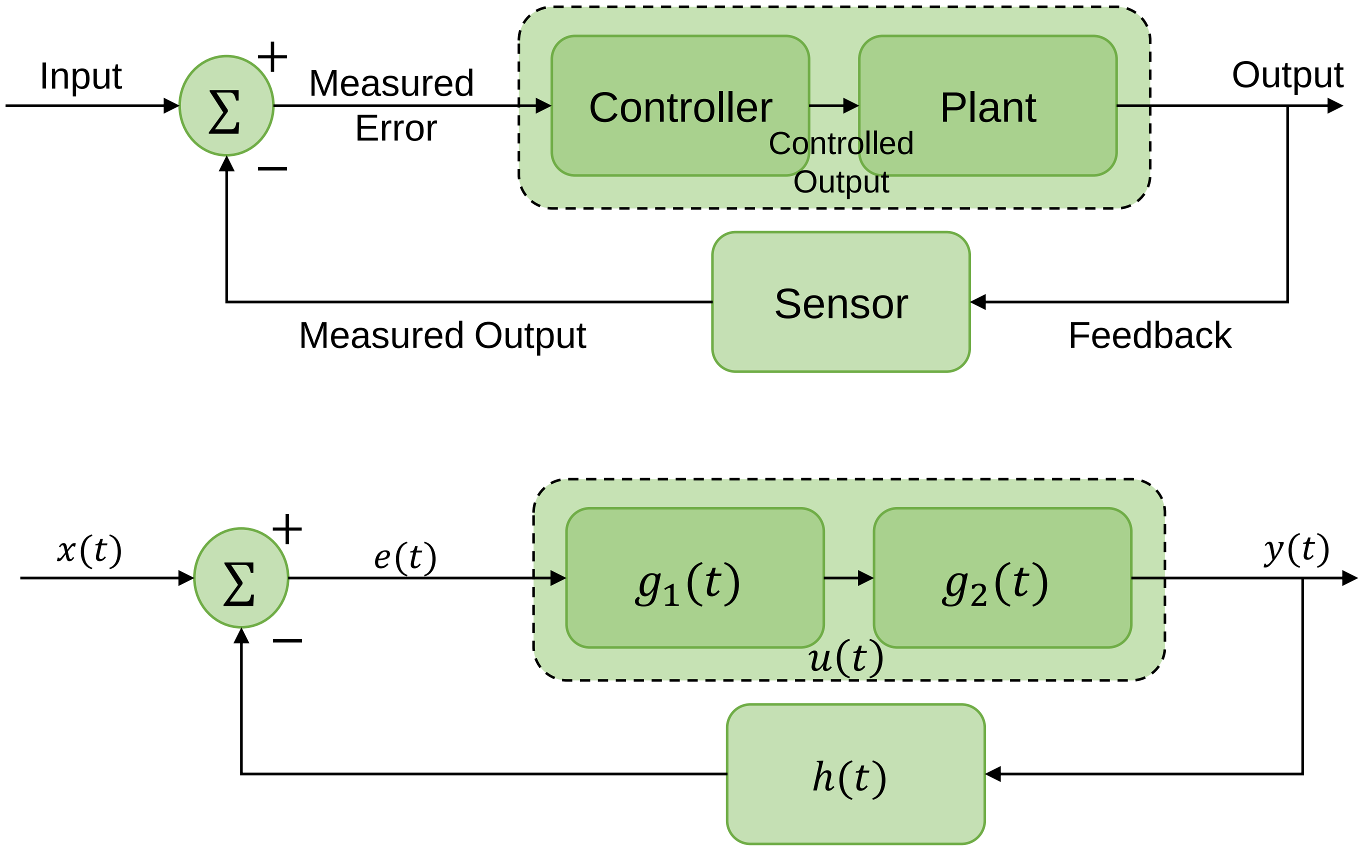
Open Loop Example



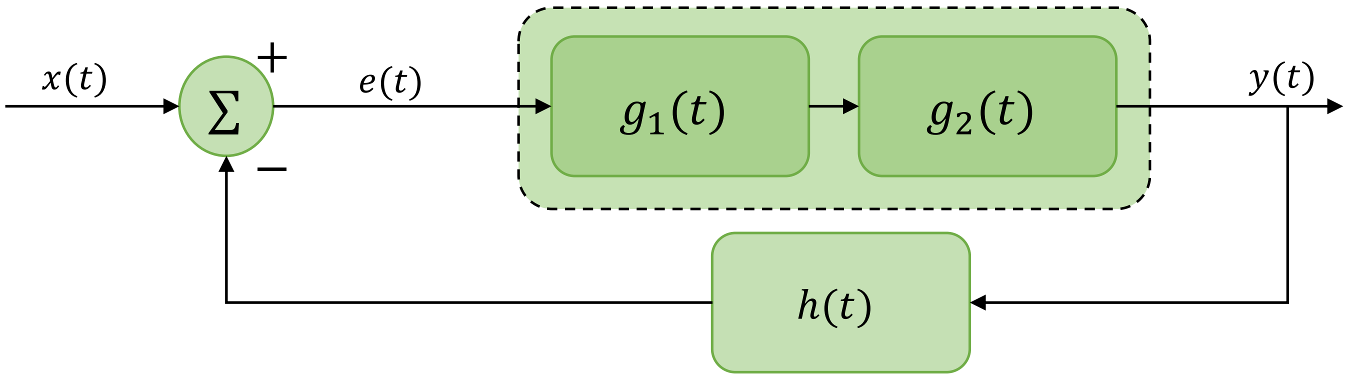
$$G(s) = \frac{s+1}{s^2+s+4}$$

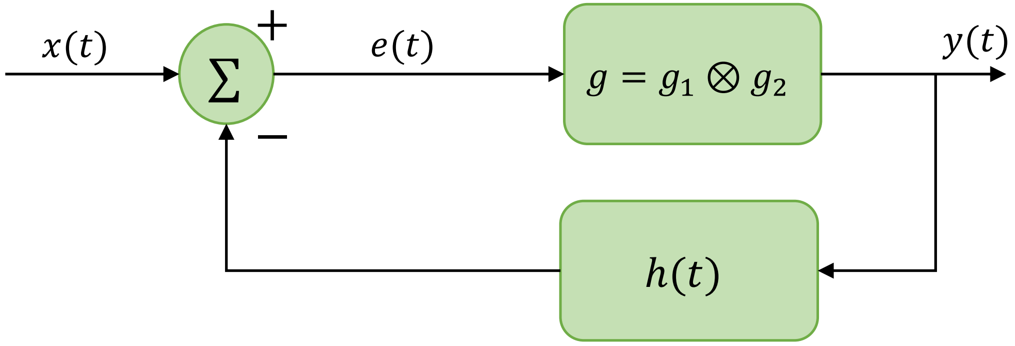
$$G_{tot}(s) = G(s)$$

Closed Loop

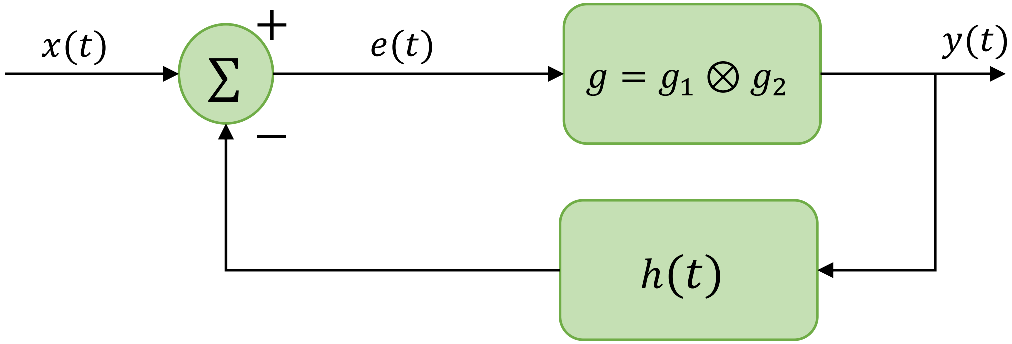

Input
Σ
+
−
Measured
Error
Controller
Controlled
Output
Plant
Output
Sensor
Measured Output
Feedback
$x(t)$
$e(t)$
$g_1(t)$
$u(t)$
$g_2(t)$
$y(t)$
$h(t)$

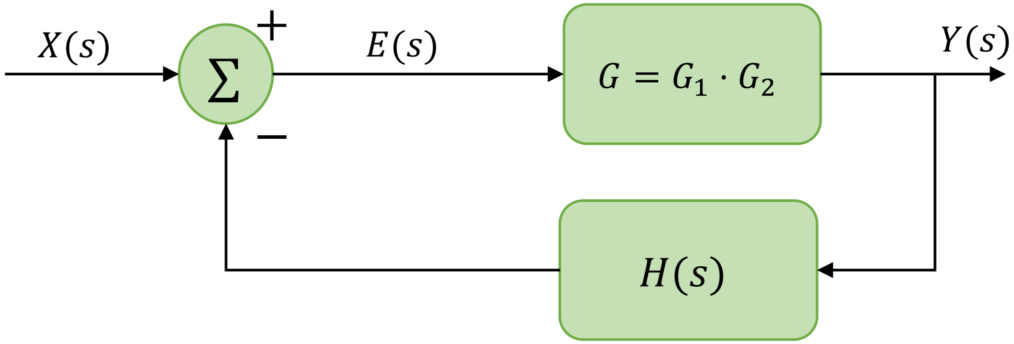
Closed Loop



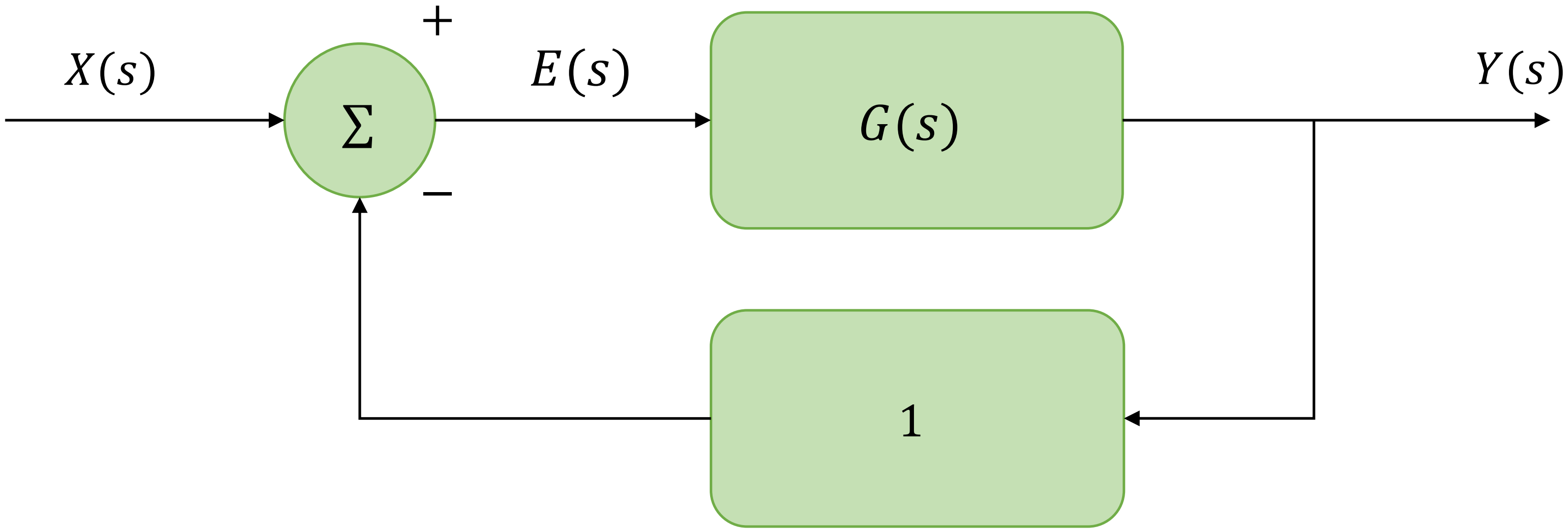


Closed Loop

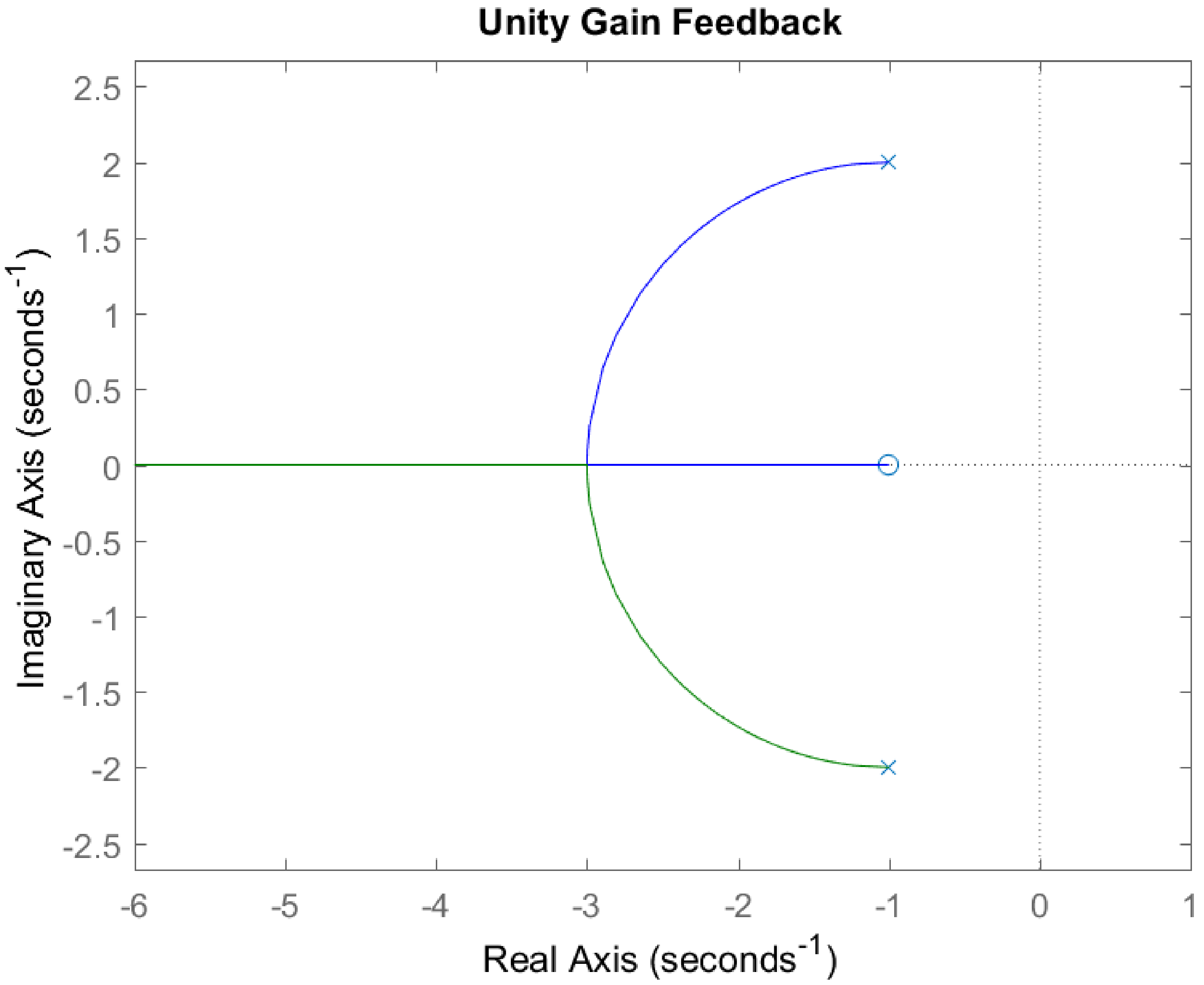




Unity Gain Feedback



Unity Gain Feedback Example

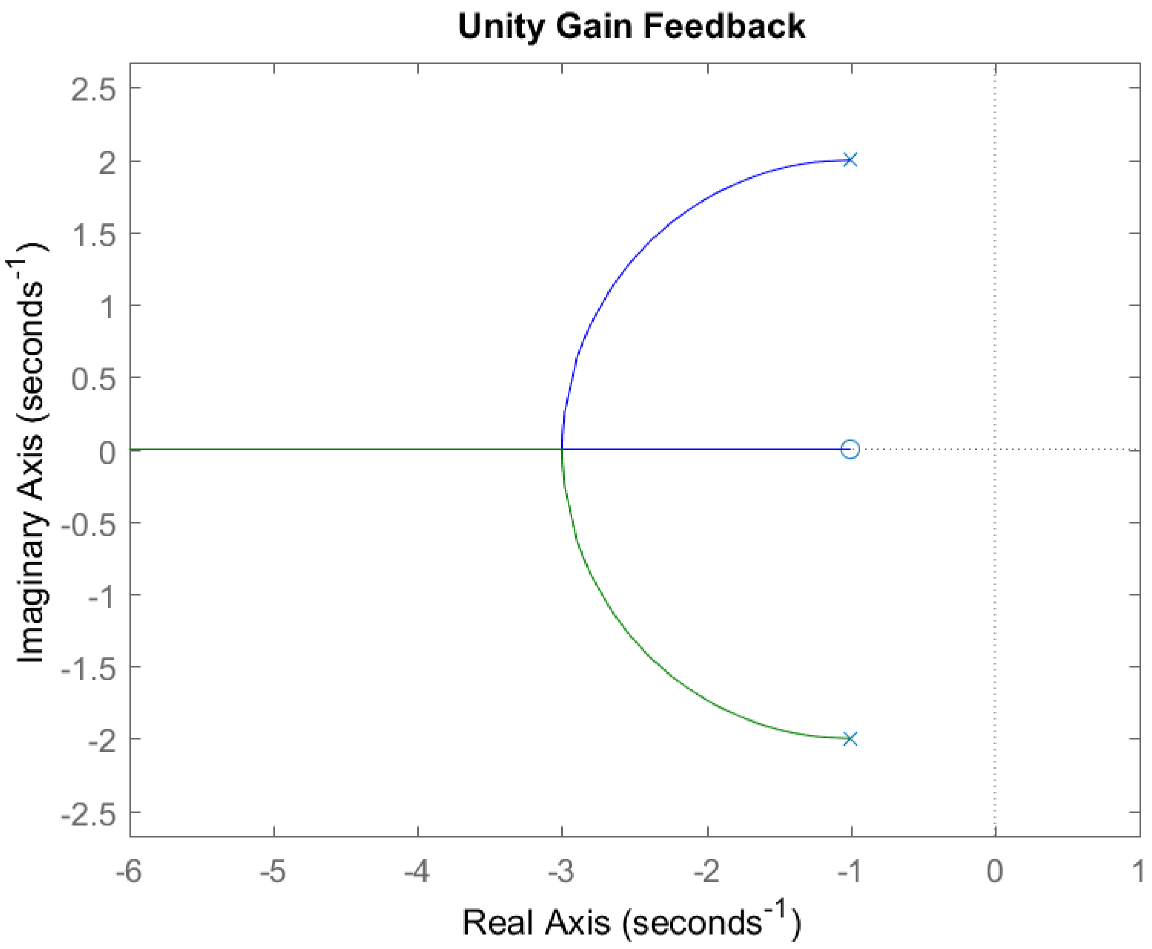


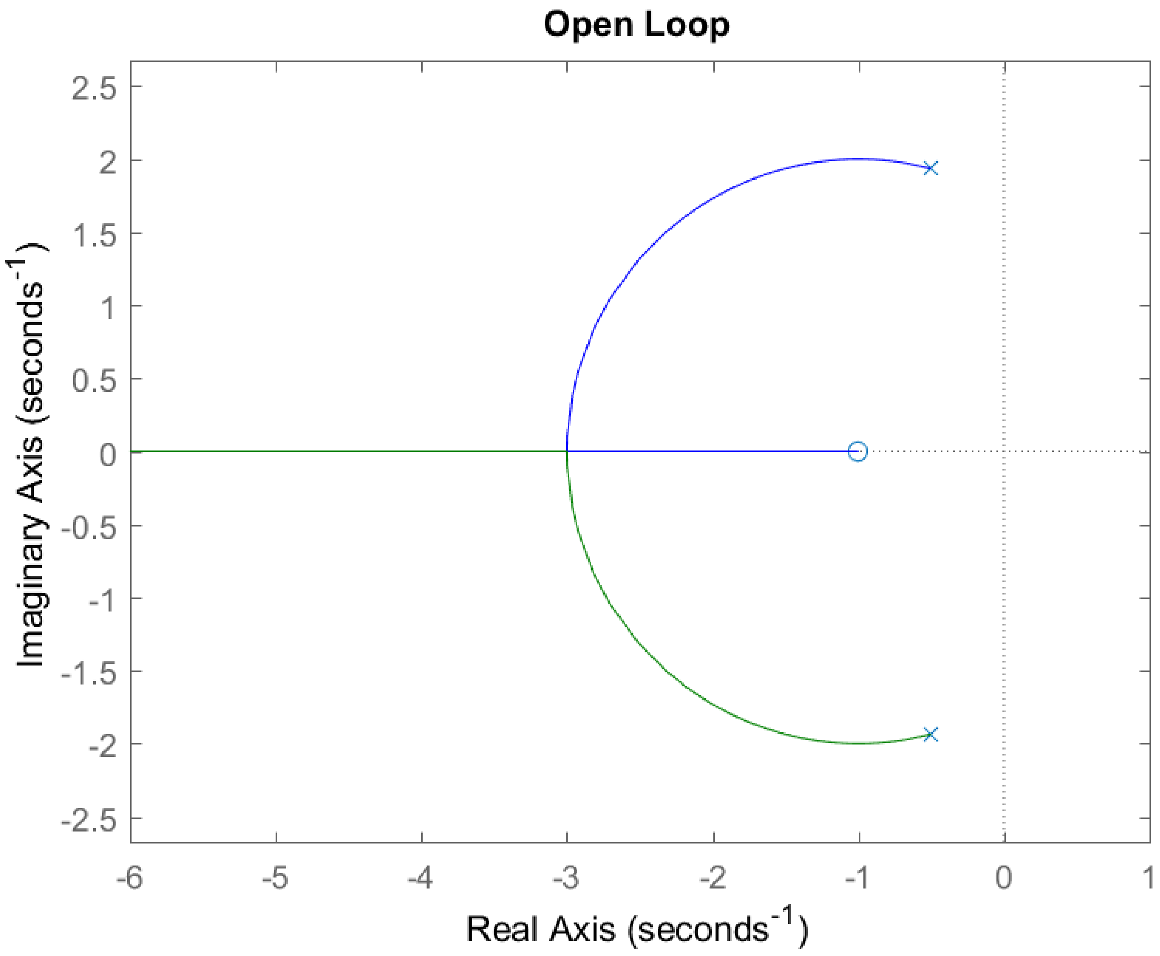
$$G(s) = \frac{s+1}{s^2+s+4}$$

$$G_{tot}(s) = \frac{G(s)}{1+G(s)}$$

$$G_{tot}(s) = \frac{s+1}{s^2+2s+5}$$

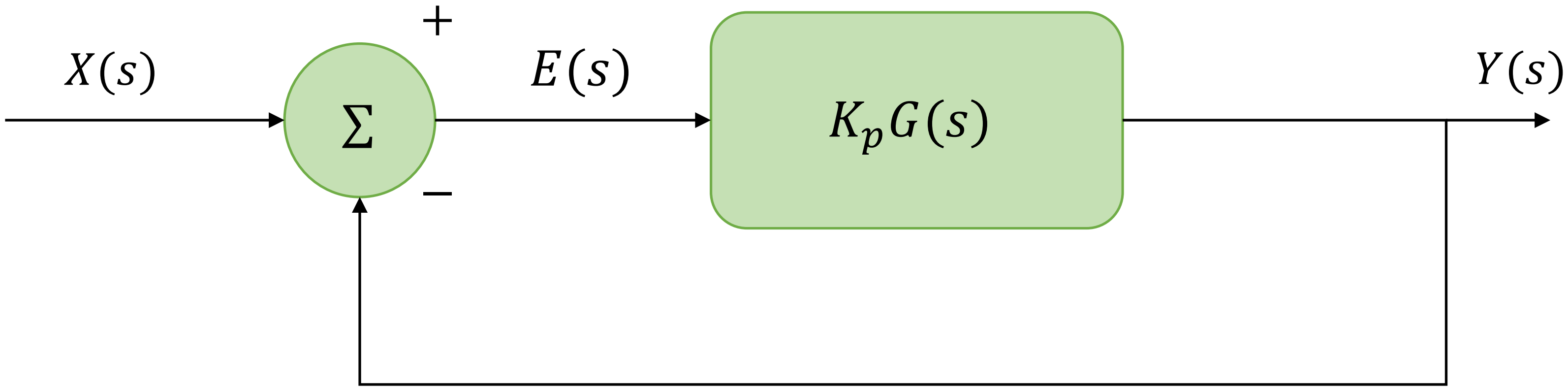
Compare with Open Loop



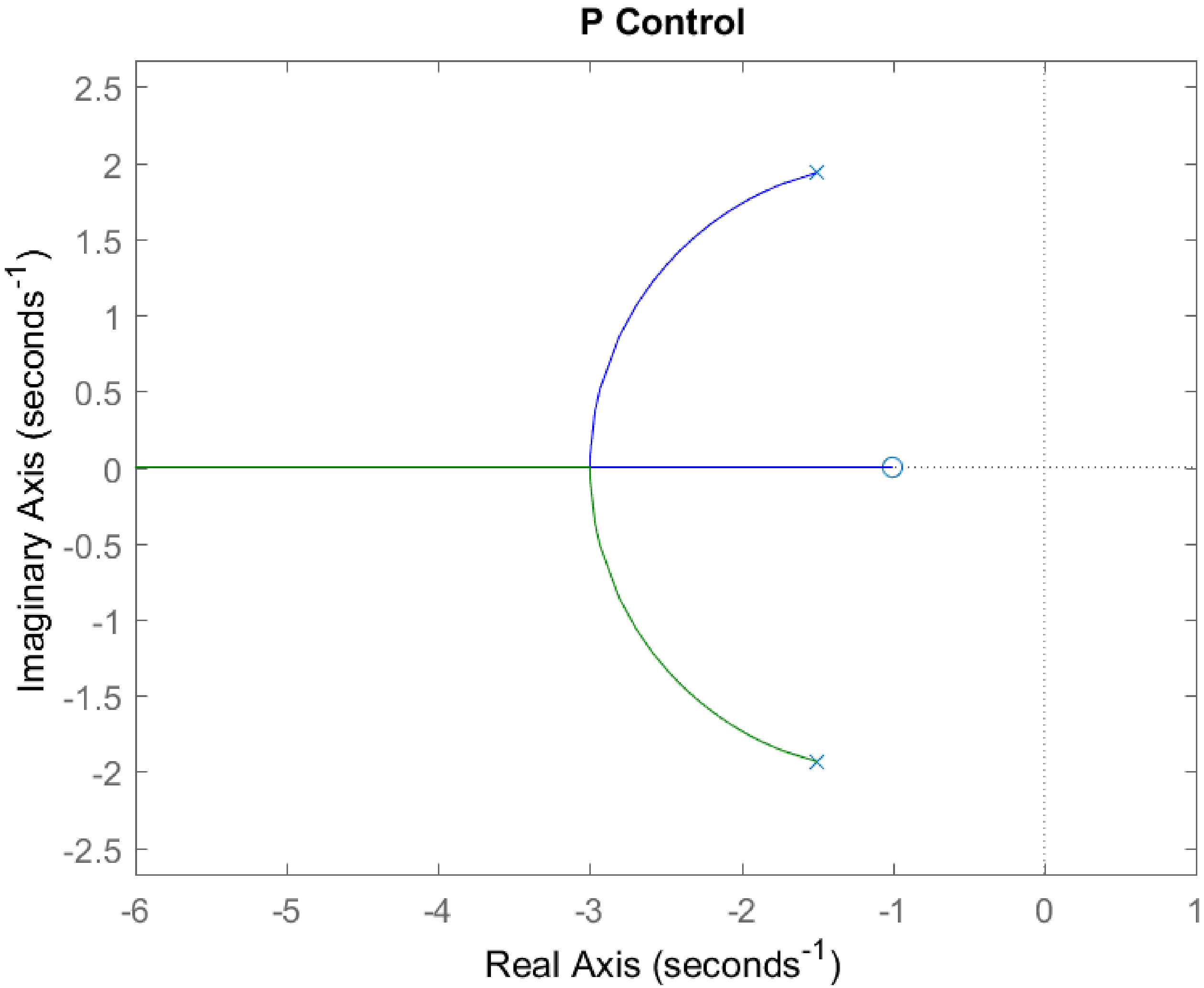


P Control

a.k.a. Proportional control



P Control Example



$$G(s) = \frac{s+1}{s^2+s+4}$$

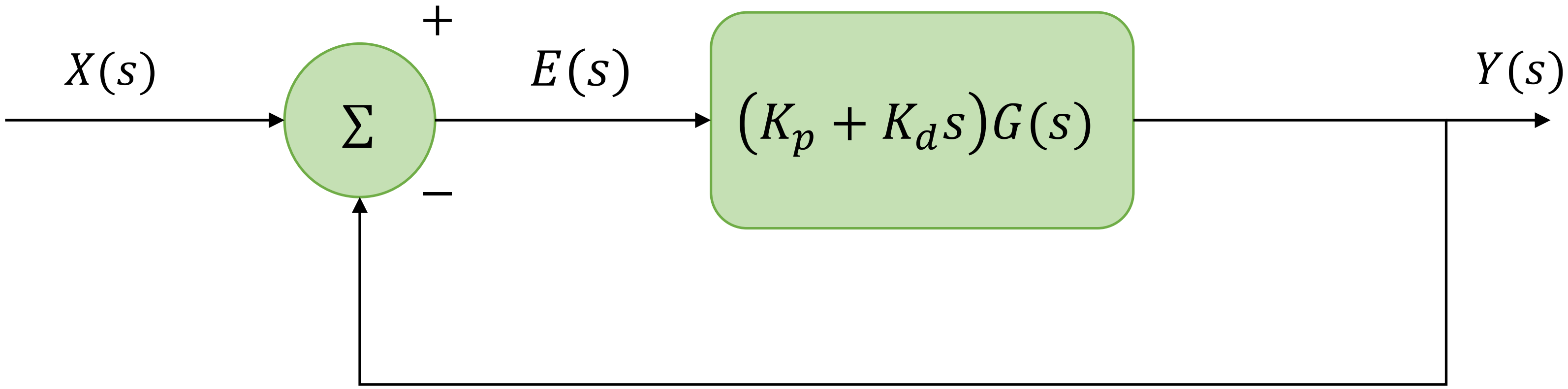
$$G_{tot}(s) = \frac{K_p G(s)}{1+K_p G(s)}$$

Let $K_p = 2$

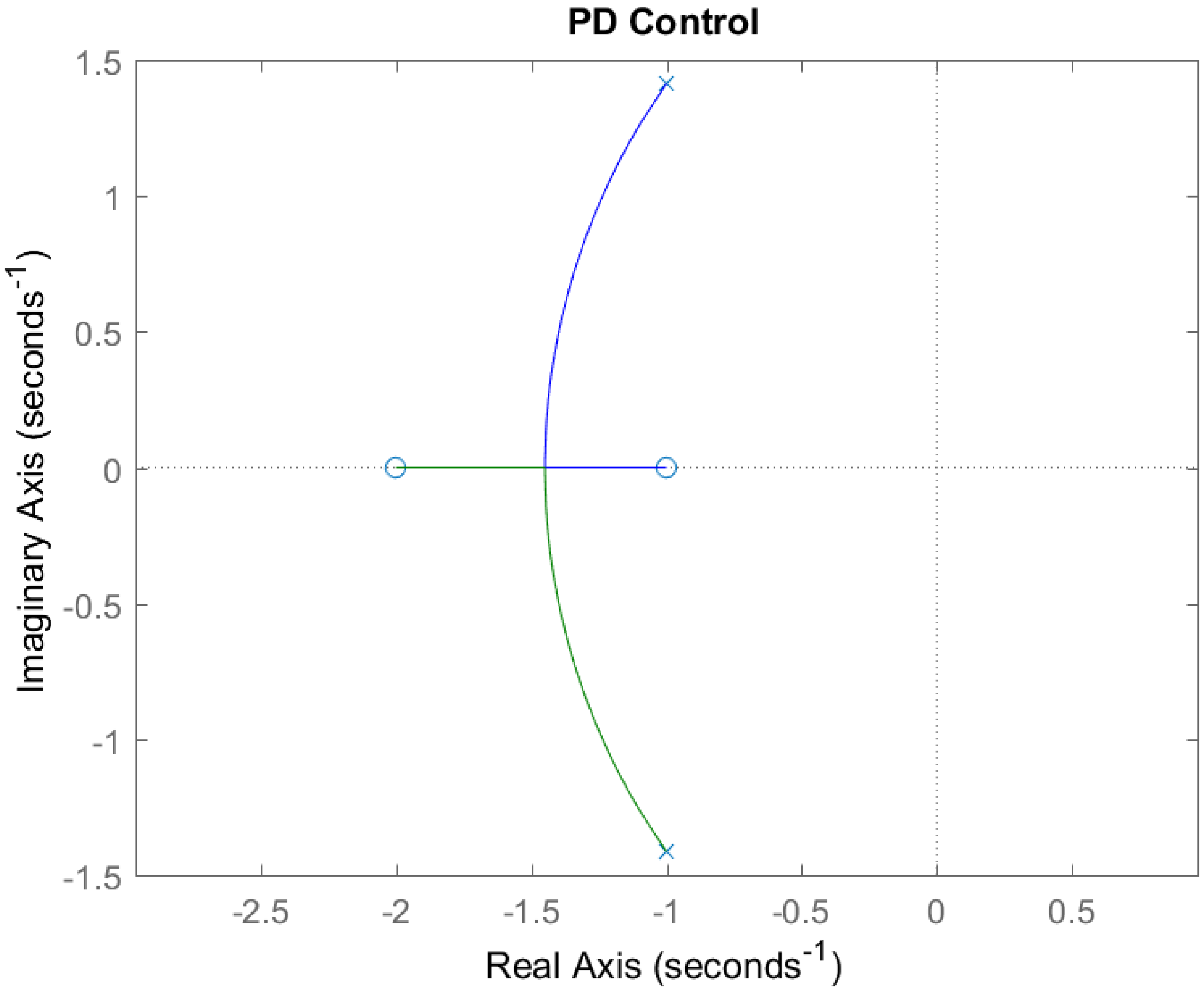
$$G_{tot}(s) = \frac{2s+2}{s^2+3s+6}$$

PD Control

a.k.a. Proportional Derivative control



PD Control Example



$$G(s) = \frac{s+1}{s^2+s+4}$$

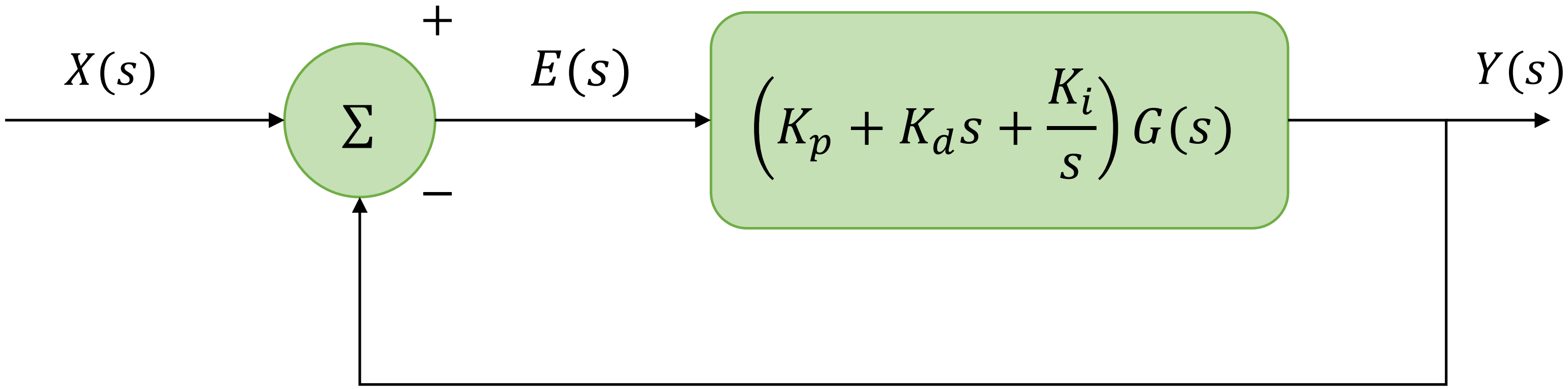
$$G_{tot}(s) = \frac{(K_p+K_d s)G(s)}{1+(K_p+K_d s)G(s)}$$

Let $K_p = 2, K_d = 1$

$$G_{tot}(s) = \frac{s^2+3s+2}{2s^2+4s+6}$$

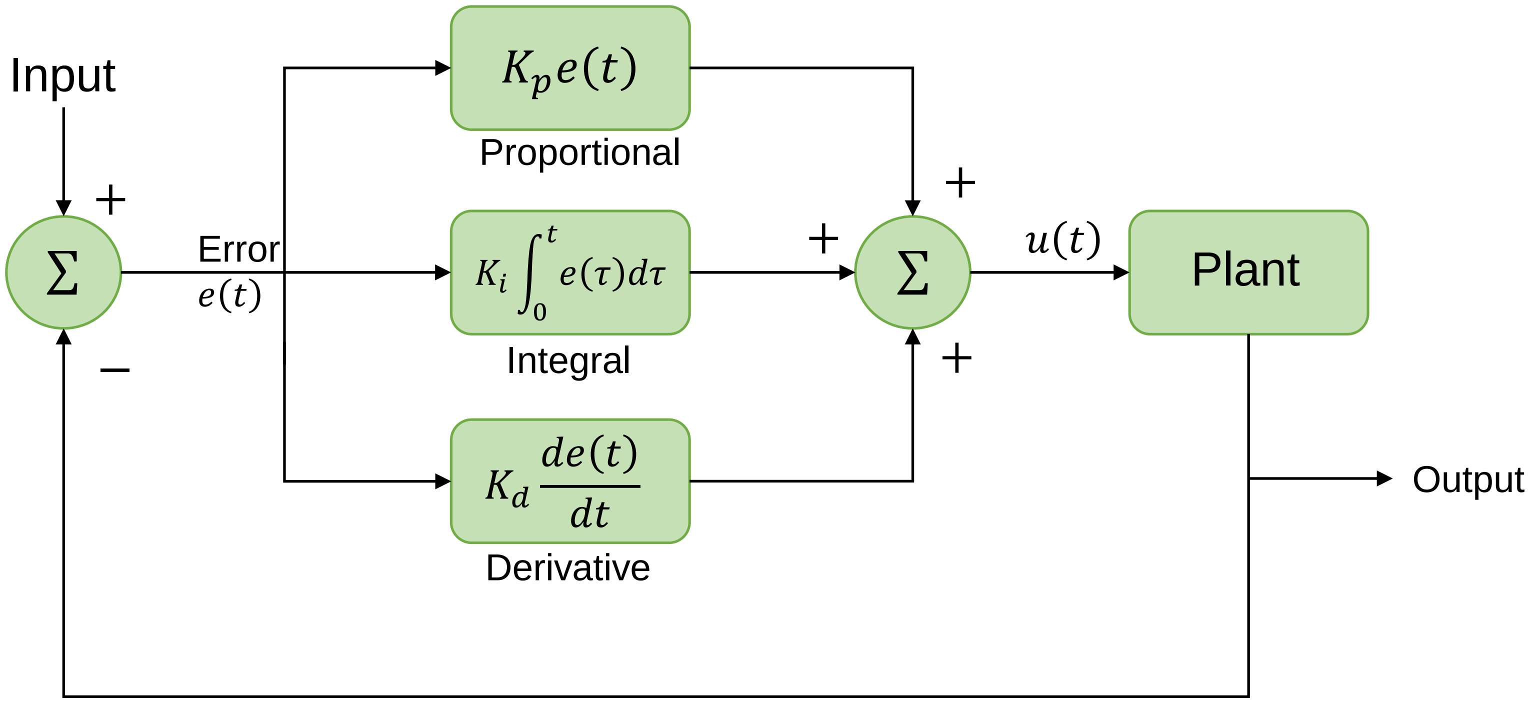
PID Control

a.k.a. Proportional Integral Derivative control

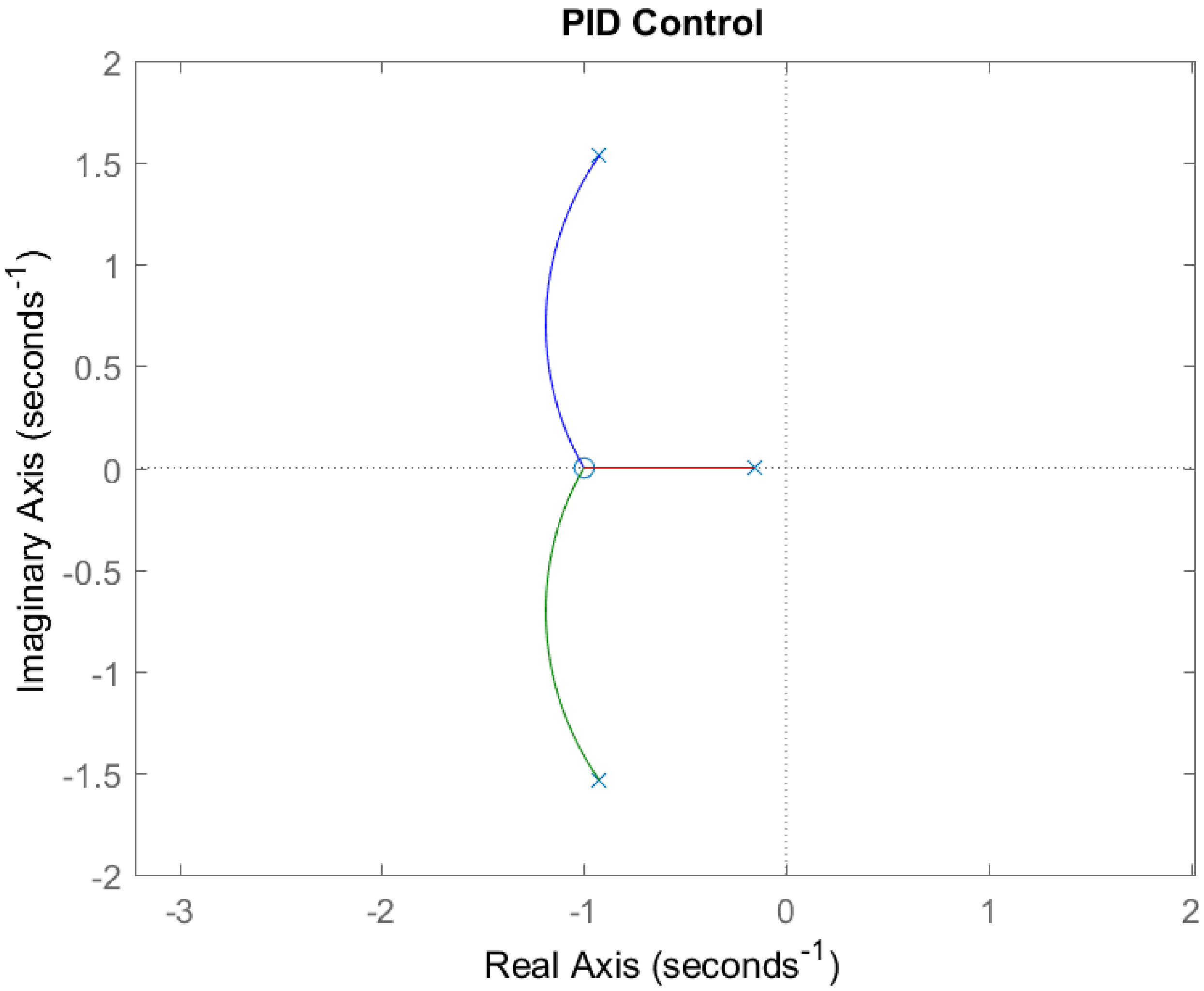


PID Control

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau \; + K_d \frac{de(t)}{dt}$$



PID Control Example



$$G(s) = \frac{s+1}{s^2+s+4}$$

$$G_{tot}(s) = \frac{\left(K_p + K_d s + \frac{K_i}{s}\right) G(s)}{1 + \left(K_p + K_d s + \frac{K_i}{s}\right) G(s)}$$

Let $K_p = 2, K_d = 1, K_i = 1$

$$G_{tot}(s) = \frac{s^3 + 3s^2 + 3s + 1}{2s^3 + 4s^2 + 7s + 1}$$

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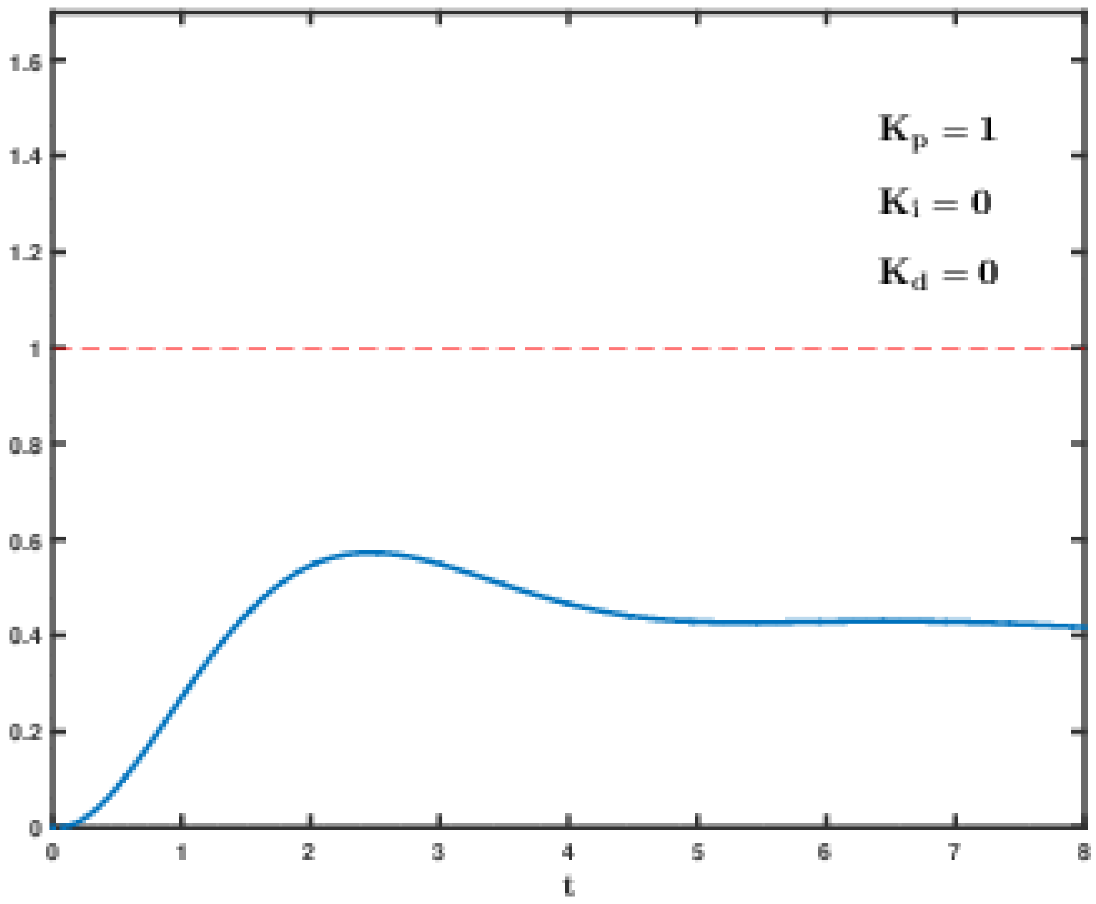
PID Control

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau \; + K_d \frac{de(t)}{dt}$$



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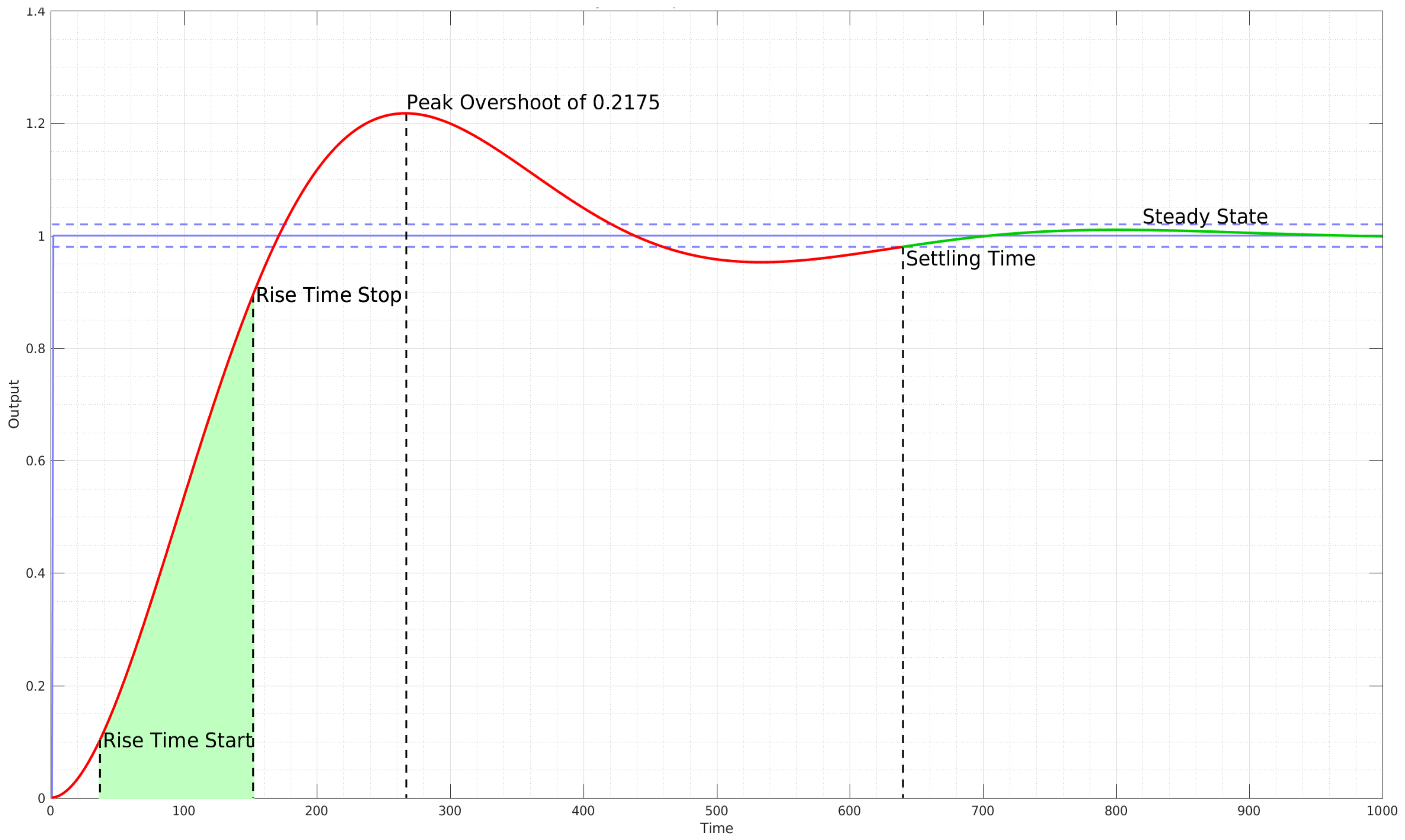
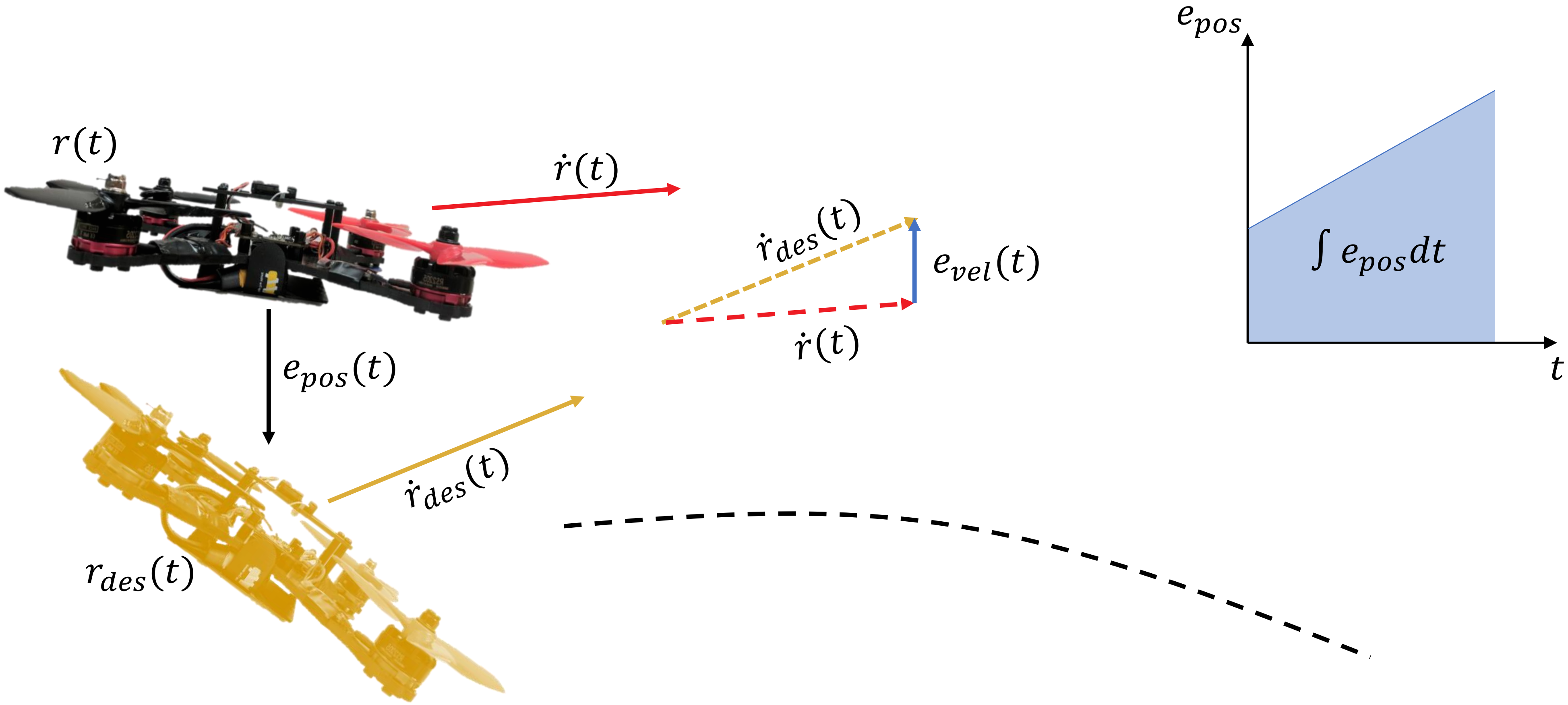
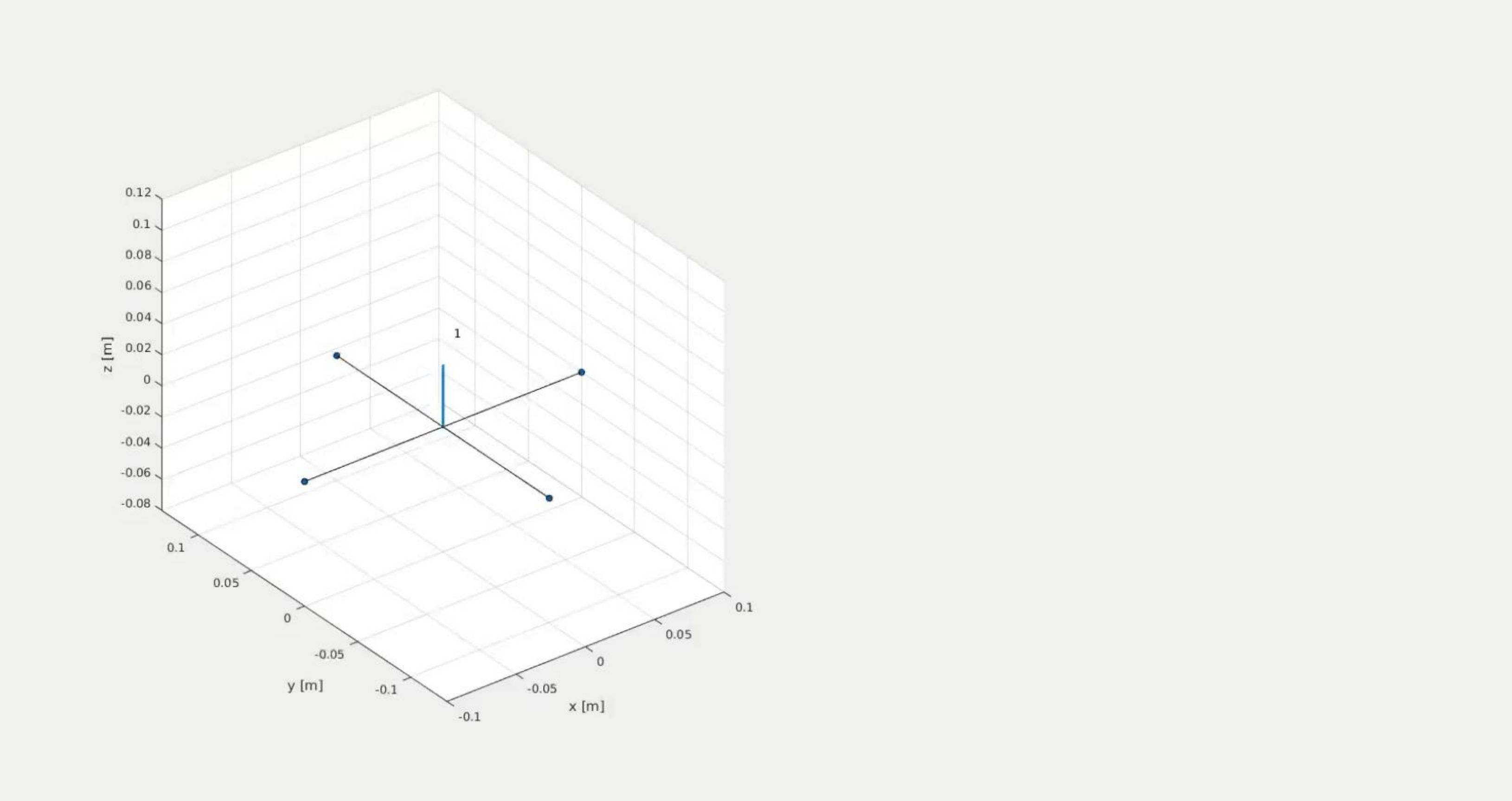
Gain Tuning


Peak Overshoot of 0.2175
Steady State
Settling Time
Rise Time Stop
Rise Time Start
Output
Time
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
700
800
900
1000

Physical Intuition

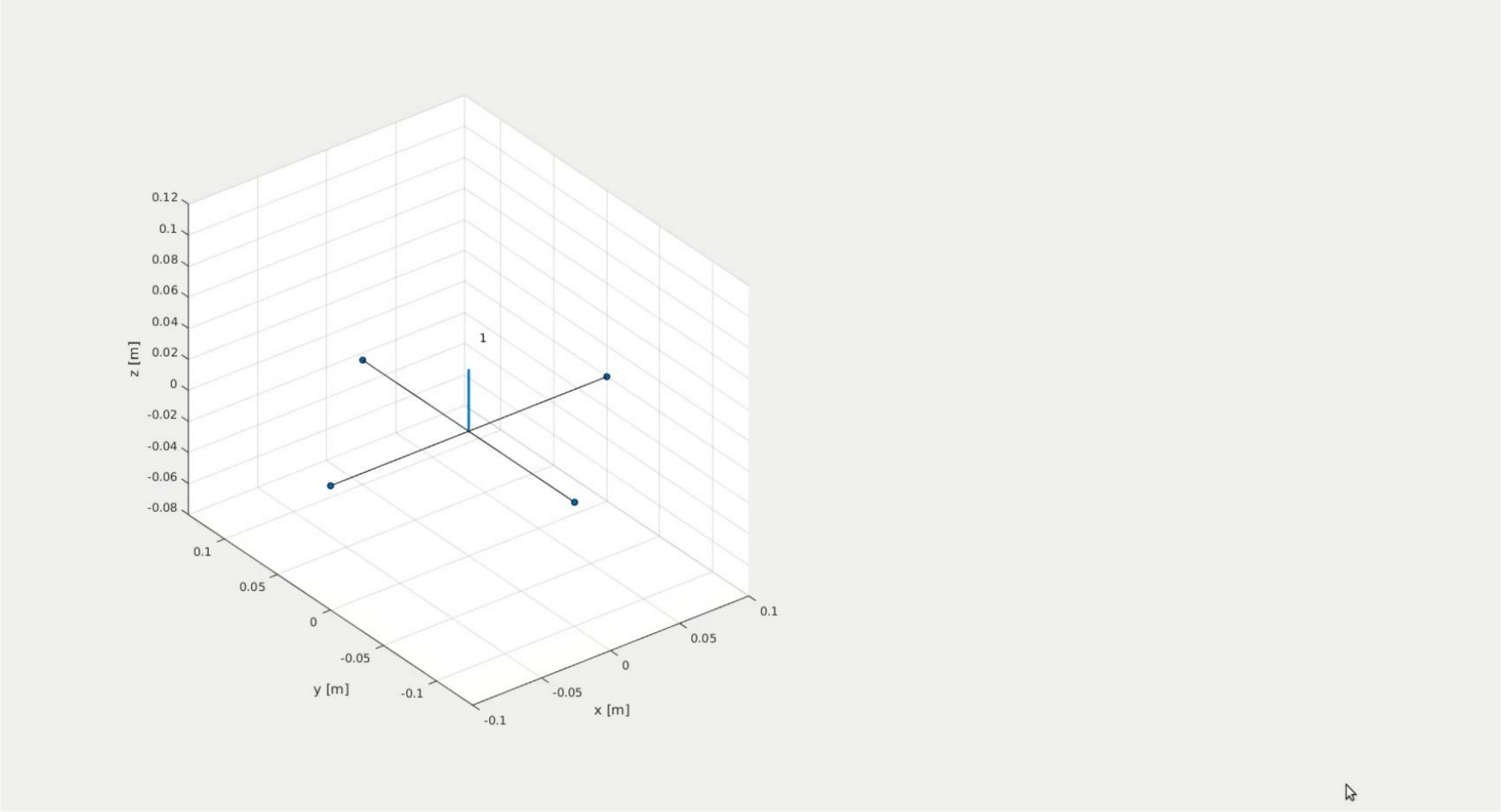


Stable

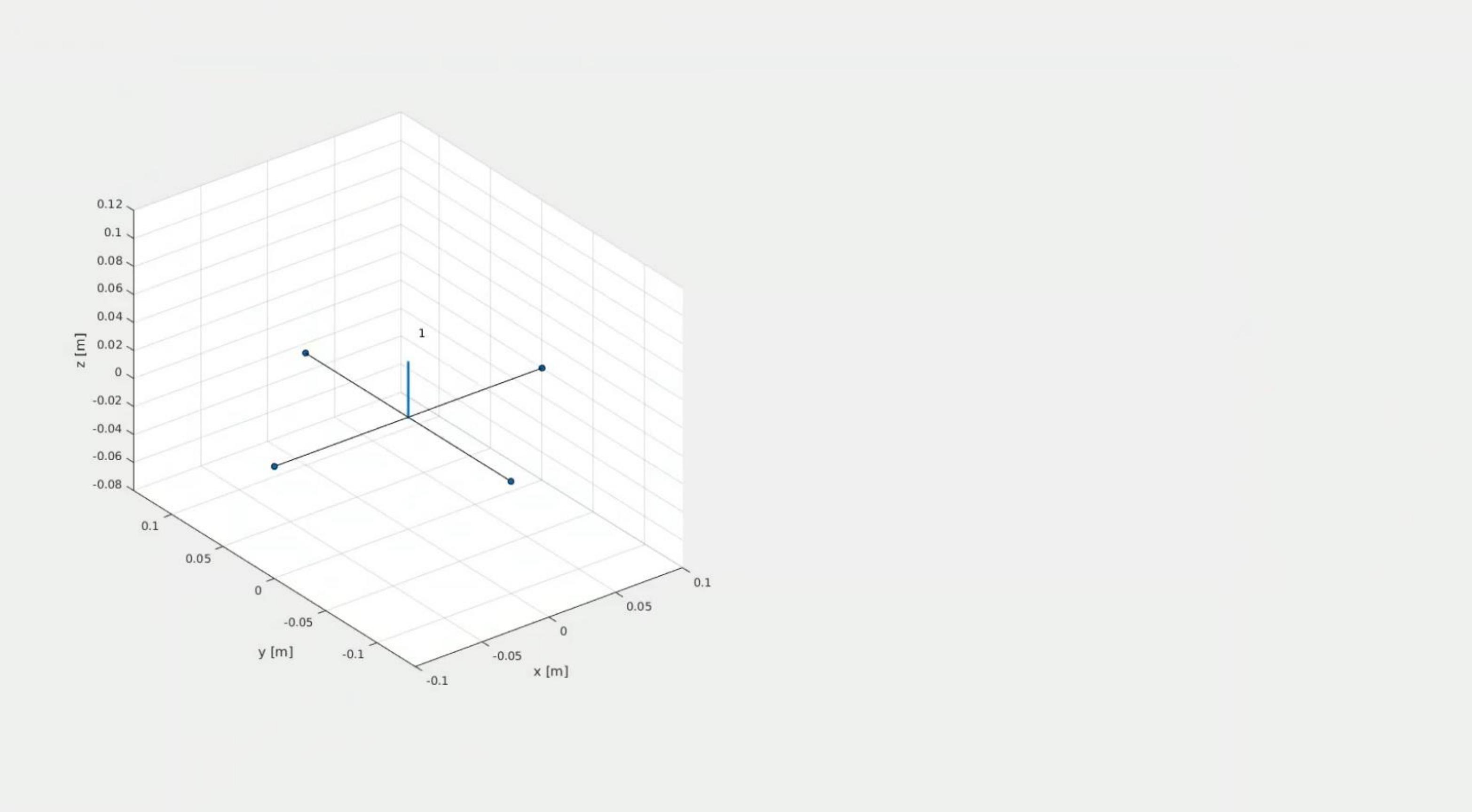


0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
z [m]
1
0.1
0.05
0
-0.05
-0.1
y [m]
-0.1
-0.05
0
0.05
0.1
x [m]

Marginally Stable



Unstable



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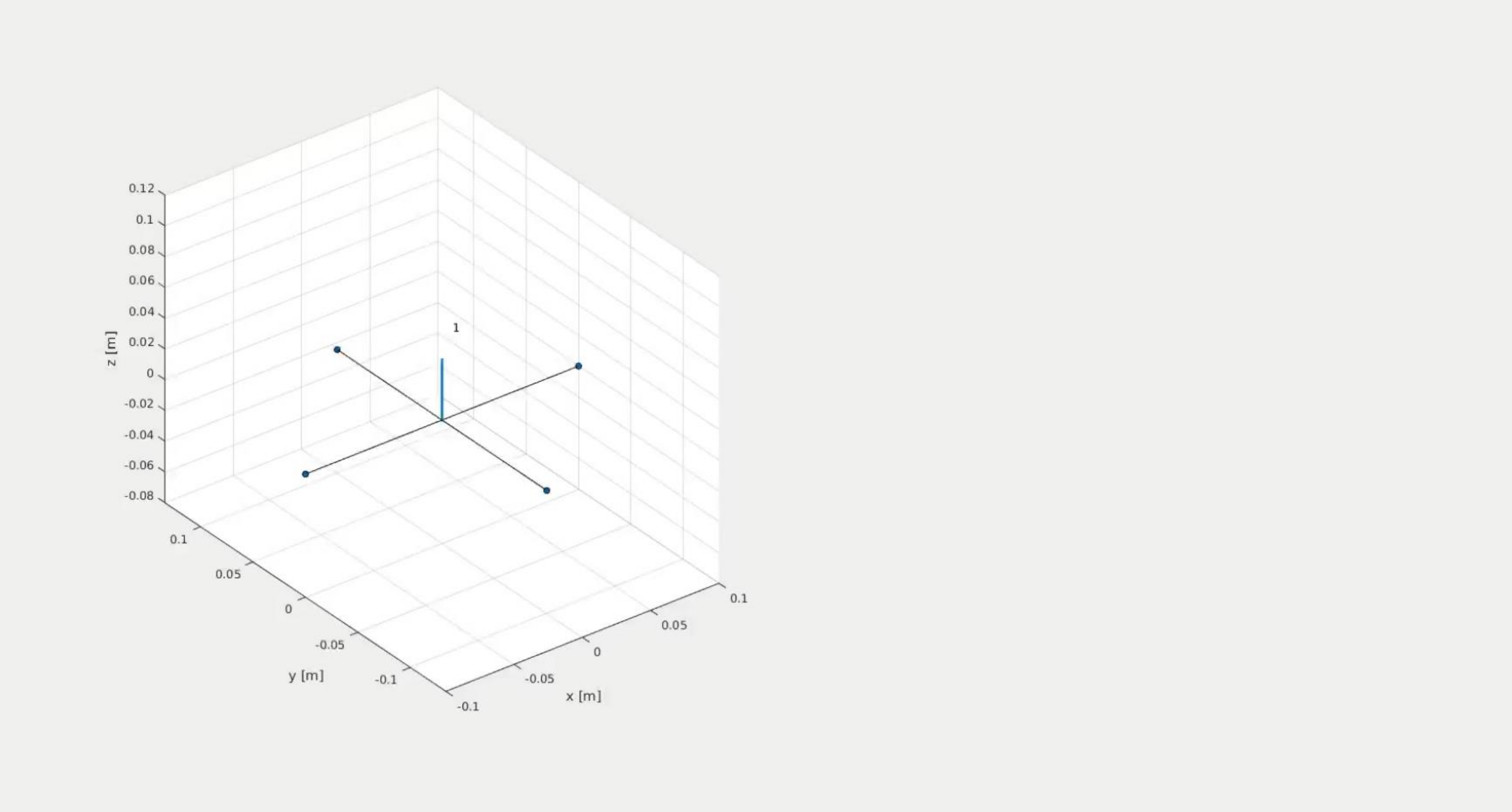
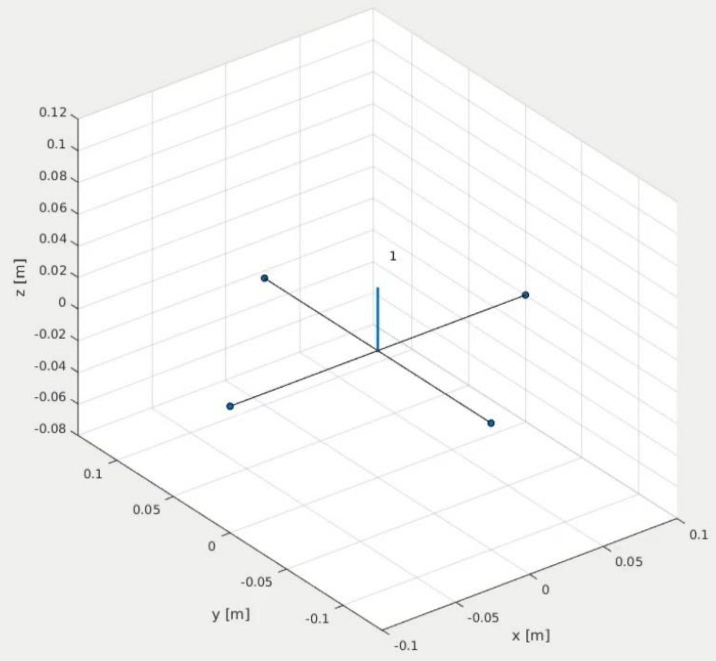
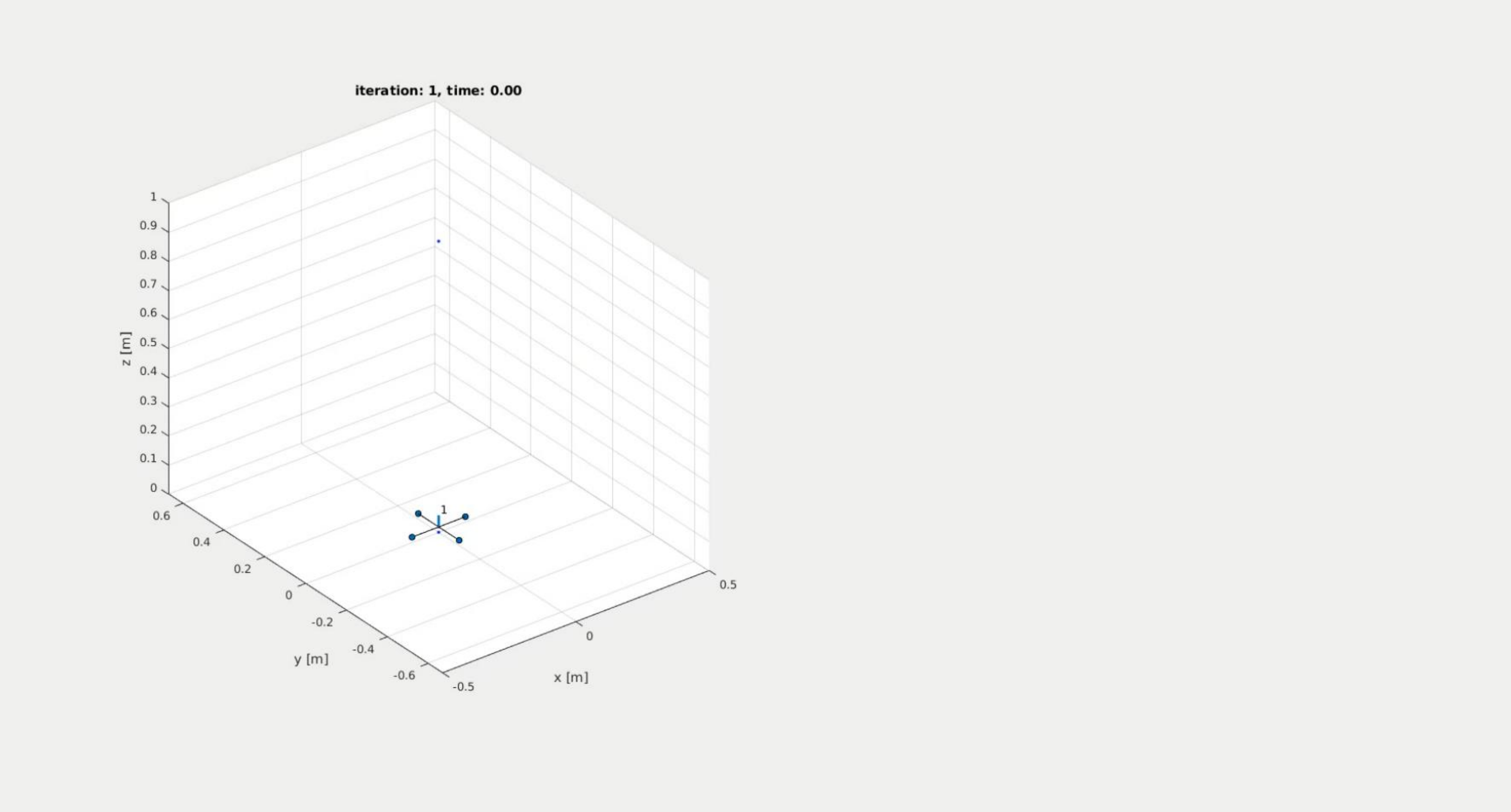
Good Gains


z [m]
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
1
y [m]
0.1
0.05
0
-0.05
-0.1
x [m]
-0.1
-0.05
0
0.05
0.1

Overdamped


0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
z [m]
1
0.1
0.05
0
-0.05
-0.1
y [m]
-0.1
-0.05
0
0.05
0.1
x [m]

Underdamped



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Manual Tuning

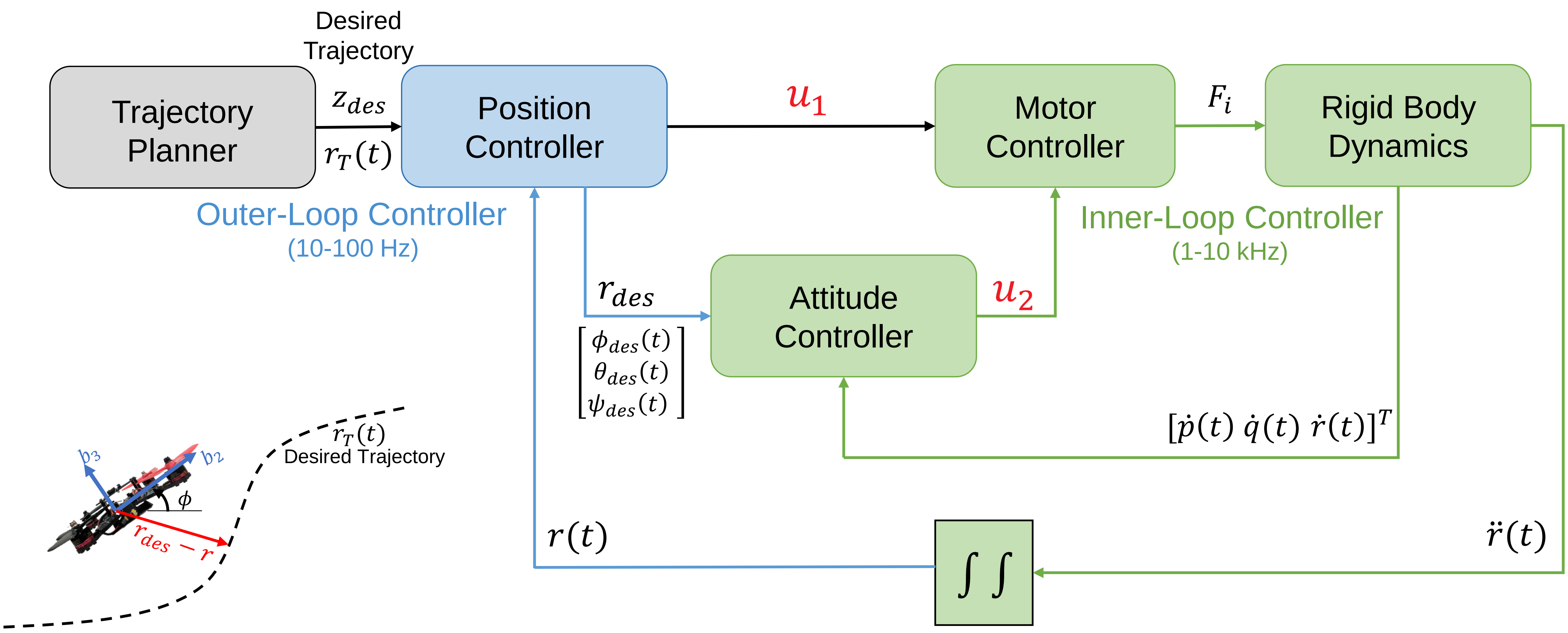
Parameter Increased	K_p	K_d	K_i
Rise Time	↓	–	↓
Peak Overshoot	↑	↓	↑
Settling Time	–	↓	↑
Steady-State Error	↓	–	Eliminate

Ziegler-Nichols Method

Control Type	K_p	$T_d = K_p/K_d$	$T_i = K_p/K_i$
P	$\frac{K_u}{2}$	-	-
PI	$\frac{9K_u}{20}$	-	$\frac{5T_u}{6}$
PD	$\frac{4K_u}{5}$	$\frac{T_u}{8}$	-
PID	$\frac{3K_u}{5}$	$\frac{T_u}{8}$	$\frac{T_u}{2}$
Some overshoot	$\frac{K_u}{3}$	$\frac{T_u}{3}$	$\frac{T_u}{2}$
No overshoot	$\frac{K_u}{5}$	$\frac{T_u}{3}$	$\frac{T_u}{2}$

K_u : Ultimate Gain, T_u : Oscillation Time Period

High Level Picture



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The Nominal Hover State

Conditions

$r = r_0$

$\theta \sim \phi \to 0 \quad \Rightarrow \cos\phi \approx \cos\theta \approx 1 \quad sin\,\phi \approx \phi$ and $sin\,\theta \approx \theta$

$\dot{r} = 0$

$\dot{\theta} = \dot{\phi} = \dot{\psi} = 0$

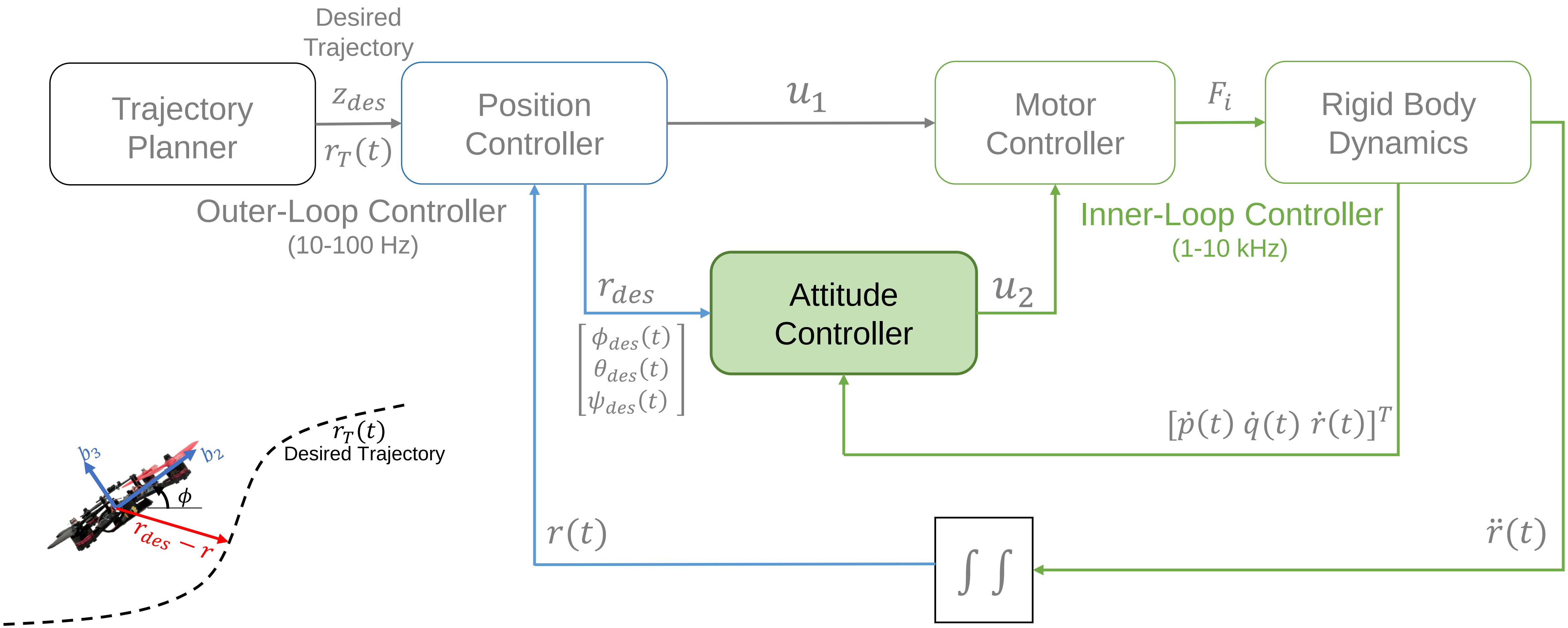
At this state, thrust F_i is given by

$F_i = \frac{mg}{4}, F_i = \mathrm{k_F}\omega_i^2$

$$\omega_i = \sqrt{\left(\frac{mg}{4\mathrm{k_F}}\right)}$$

Euler Notation: ZXY

High Level Picture



Recall Angular Velocity

Recall $\omega^b = R^T \dot{R}$

For the ZXY Euler angles: (ψ, ϕ, θ)

$$\omega^b = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c_\theta & 0 & -c_\phi s_\theta \\ 0 & 1 & s_\phi \\ s_\theta & 0 & c_\phi c_\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

Roll Rate
Pitch Rate
Yaw Rate

Body Frame

World/Inertial Frame

Attitude Control

Recall Euler's equation,

$$I\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix}p\\ q\\ r\end{bmatrix} \times I \begin{bmatrix}p\\ q\\ r\end{bmatrix}$$, p, q, r are angular velocities w.r.t. x, y, z direction

Now, assuming xy symmetric quadrotor and a diagonal moment of Inertia matrix $I \left(I_{xx} = I_{yy} \right)$

$I_{xx}\dot{p} = u_{2,x} - q\,r\left(I_{zz} - I_{yy}\right)$
$I_{yy}\dot{q} = u_{2,y} - p\,r(I_{xx} - I_{zz})$
$I_{zz}\dot{r} = u_{2,z}$

Assuming angular velocity in the z_B direction $(r \approx 0)$ is small

$\dot{p} = \frac{u_{2,x}}{I_{xx}}; \dot{q} = \frac{u_{2,y}}{I_{yy}}; \dot{r} = \frac{u_{2,z}}{I_{zz}}$

Attitude Control

Recall $\gamma = \frac{k_F}{k_M}$

$$\dot{p} = \frac{u_{2,x}}{I_{xx}} = \frac{L}{I_{xx}}(F_2 - F_4)$$
$$\dot{q} = \frac{u_{2,y}}{I_{yy}} = \frac{L}{I_{yy}}(F_3 - F_1)$$
$$\dot{r} = \frac{u_{2,z}}{I_{zz}} = \frac{\gamma}{I_{zz}}(F_1 - F_2 + F_3 - F_4)$$

Near nominal hover state, the PD control law can be given by

$$u_2 = \begin{bmatrix} \dot{p}_{des} + k_{p,\phi}(\phi_{des} - \phi) + k_{d,\phi}(p_{des} - p) \\ \dot{q}_{des} + k_{p,\theta}(\theta_{des} - \theta) + k_{d,\theta}(q_{des} - q) \\ \dot{r}_{des} + k_{p,\psi}(\psi_{des} - \psi) + k_{d,\psi}(r_{des} - r) \end{bmatrix}$$

Attitude Control

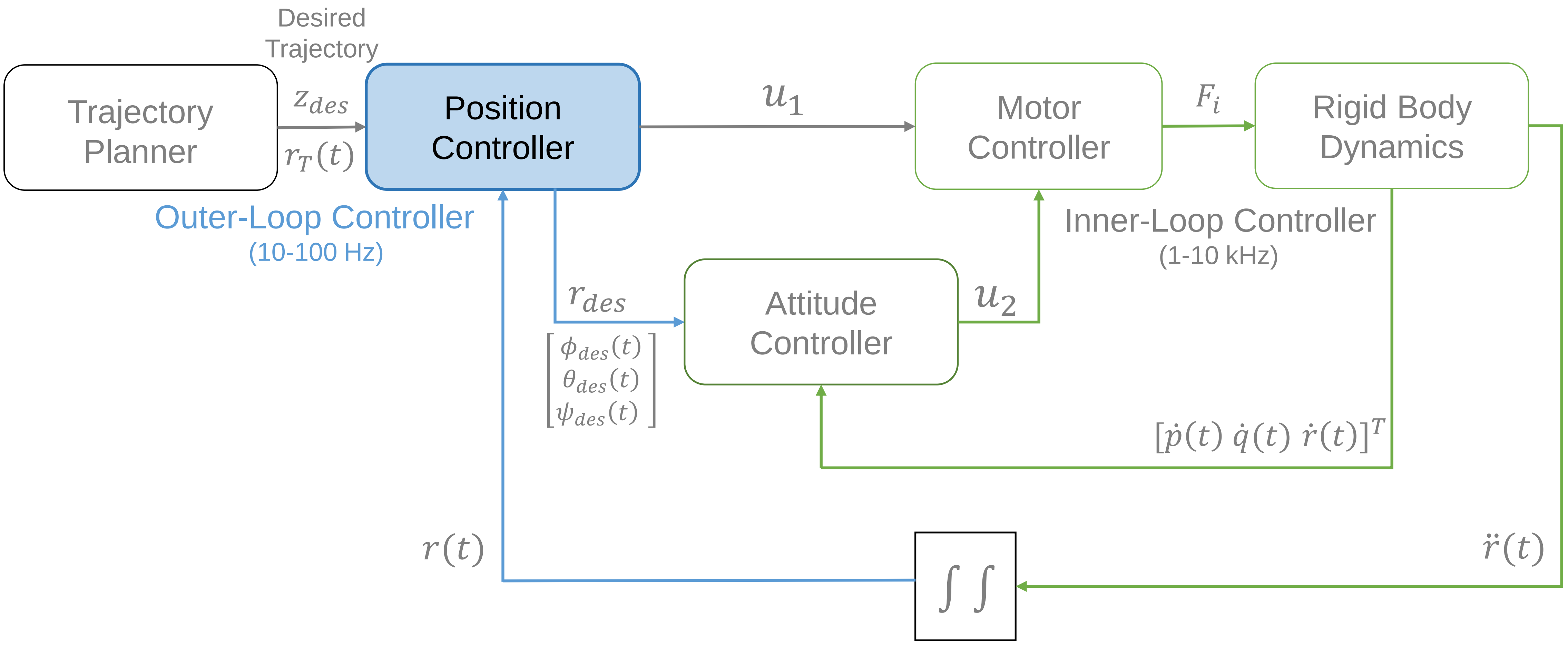
The input u_{des} can be calculated using

Recall $\gamma = \frac{k_F}{k_M}$

$$u_{des} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & L & 0 & -L \\ -L & 0 & L & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$$u_{des} = \begin{bmatrix} k_F & k_F & k_F & k_F \\ 0 & k_F L & 0 & -k_F L \\ -k_F L & 0 & k_F L & 0 \\ k_M & -k_M & k_M & -k_M \end{bmatrix} \begin{bmatrix} \omega_{1,des}^2 \\ \omega_{2,des}^2 \\ \omega_{3,des}^2 \\ \omega_{4,des}^2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

High Level Picture



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Position Control

Hover Controller

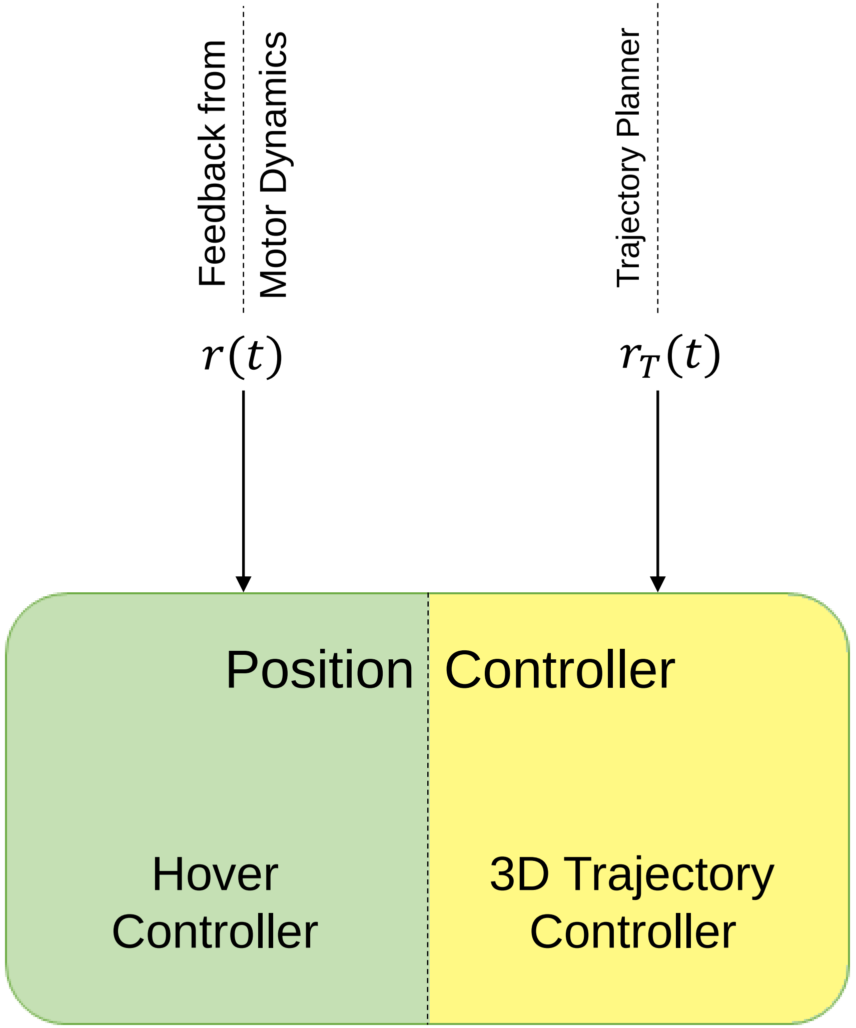
Maintains the position at a desired x, y, z

u_1 controls position along z_A
$u_{2,x}$ and $u_{2,y}$ along controls roll and pitch angle

$u_{2,z}$ controls yaw angle

3D Trajectory Controller

Tracks the trajectory



A: World Frame

Position Control

Hover Controller

Let $\begin{bmatrix} r_T(t) \\ \psi_T(t) \end{bmatrix}$ be the trajectory and yaw angle we want to track

Let us assume that our yaw remains fixed,

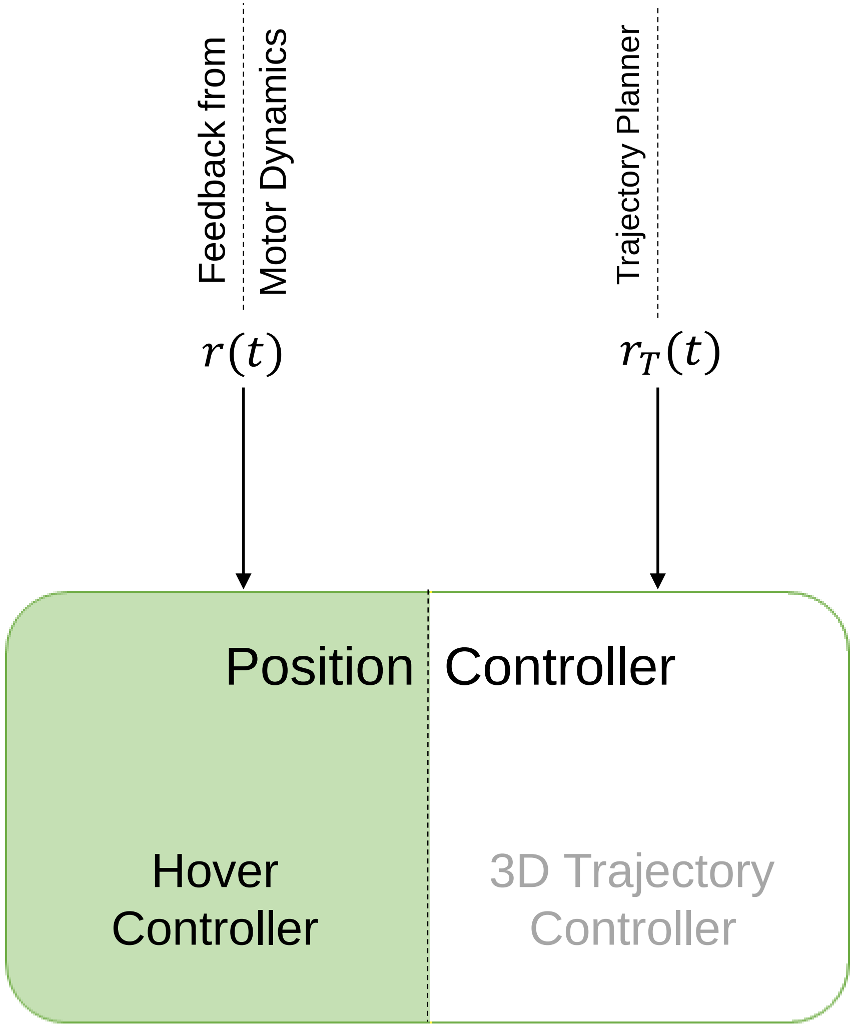
$\psi_T(t) = \psi_0$

PID feedback of position error $(e_i = r_{i,T} - r_i)$ to calculate $\ddot{r}_i^{des}$

$$(\ddot{r}_{i,T} - \ddot{r}_i^{des}) + k_{D,i}(\dot{r}_{i,T} - \dot{r}_i) + k_{P,i}(r_{i,T} - r_i) + k_{I,i}\int (r_{i,T} - r_i) = 0$$

where $i \in \{x, y, z\}$

For hover, $\dot{r}_{i,T} = \ddot{r}_{i,T} = 0$



Position Control

Hover Controller

Recall Newton's Equation of motion

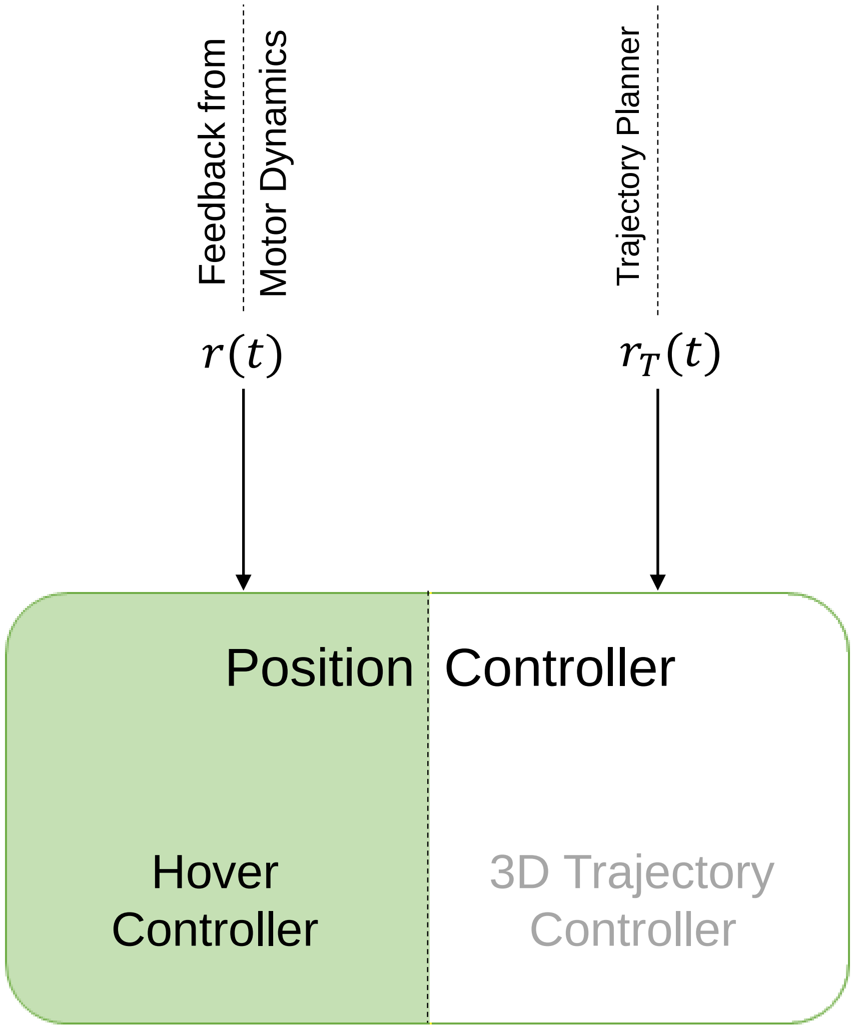
$$m\ddot{r} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + {}^{A}R_{B} \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$$

Now, linearizing the equation, we can say

$$\ddot{r}_{1,des} = g(\Delta\theta_{des}\, cos\psi_T + \Delta\phi_{des}\, sin\psi_T)$$

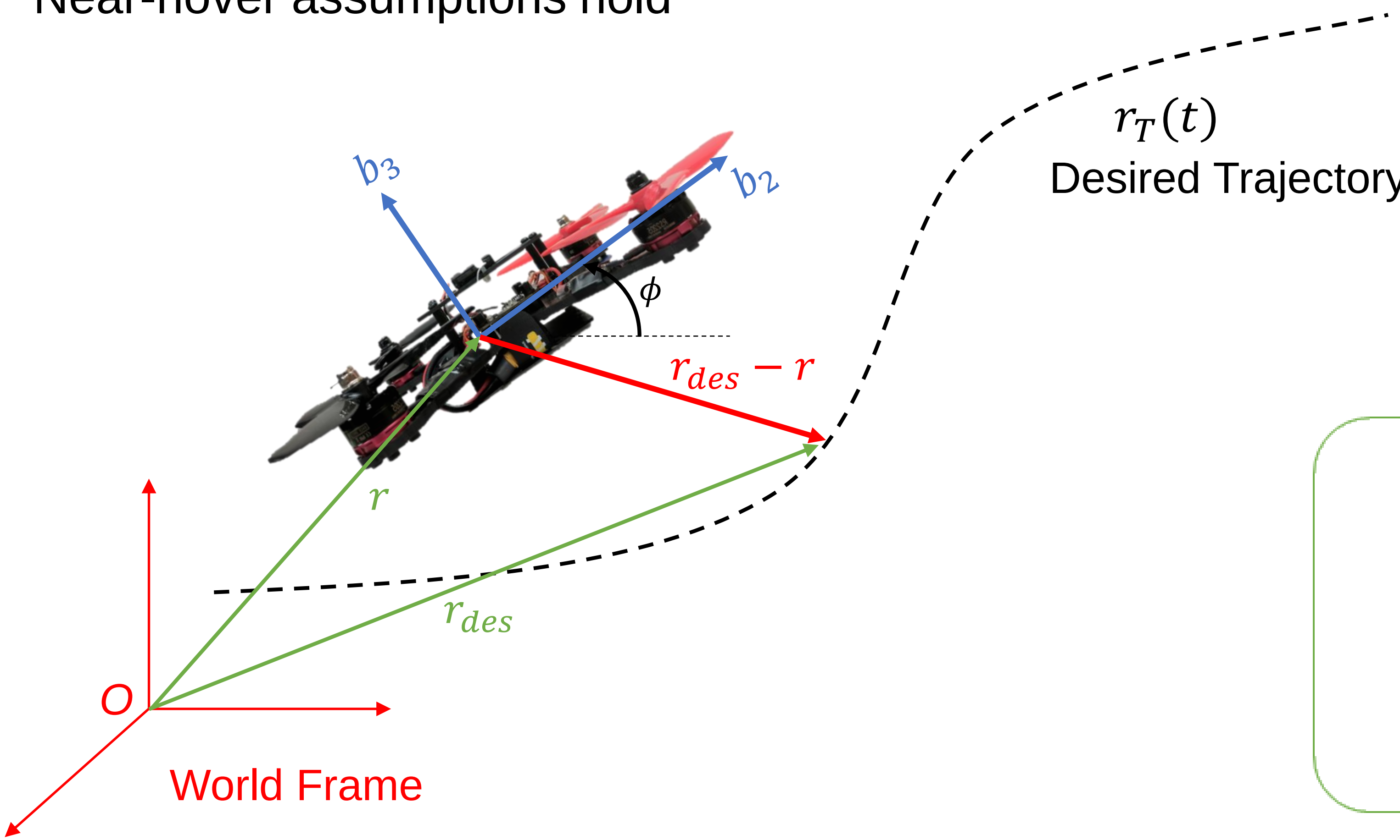
$$\ddot{r}_{2,des} = g(\Delta\theta_{des}\, sin\psi_T - \Delta\phi_{des}\, cos\psi_T)$$

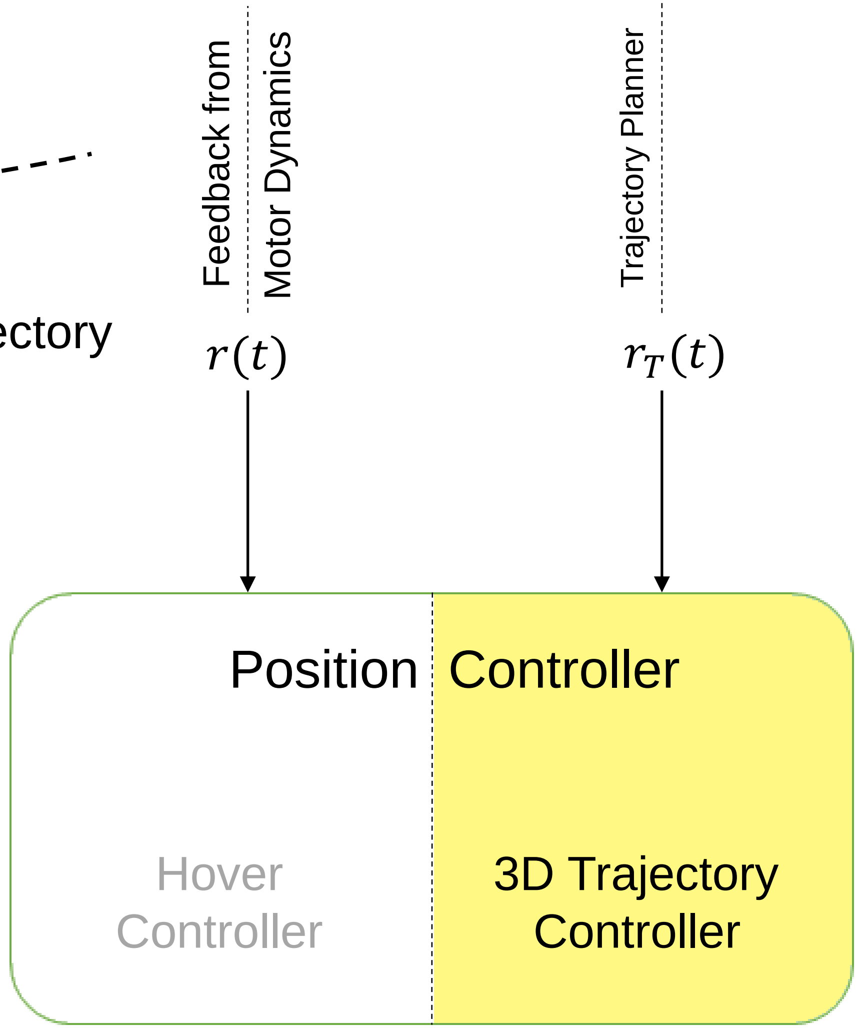
$$\ddot{r}_{3,des} = \frac{u_{1,des}}{m}$$



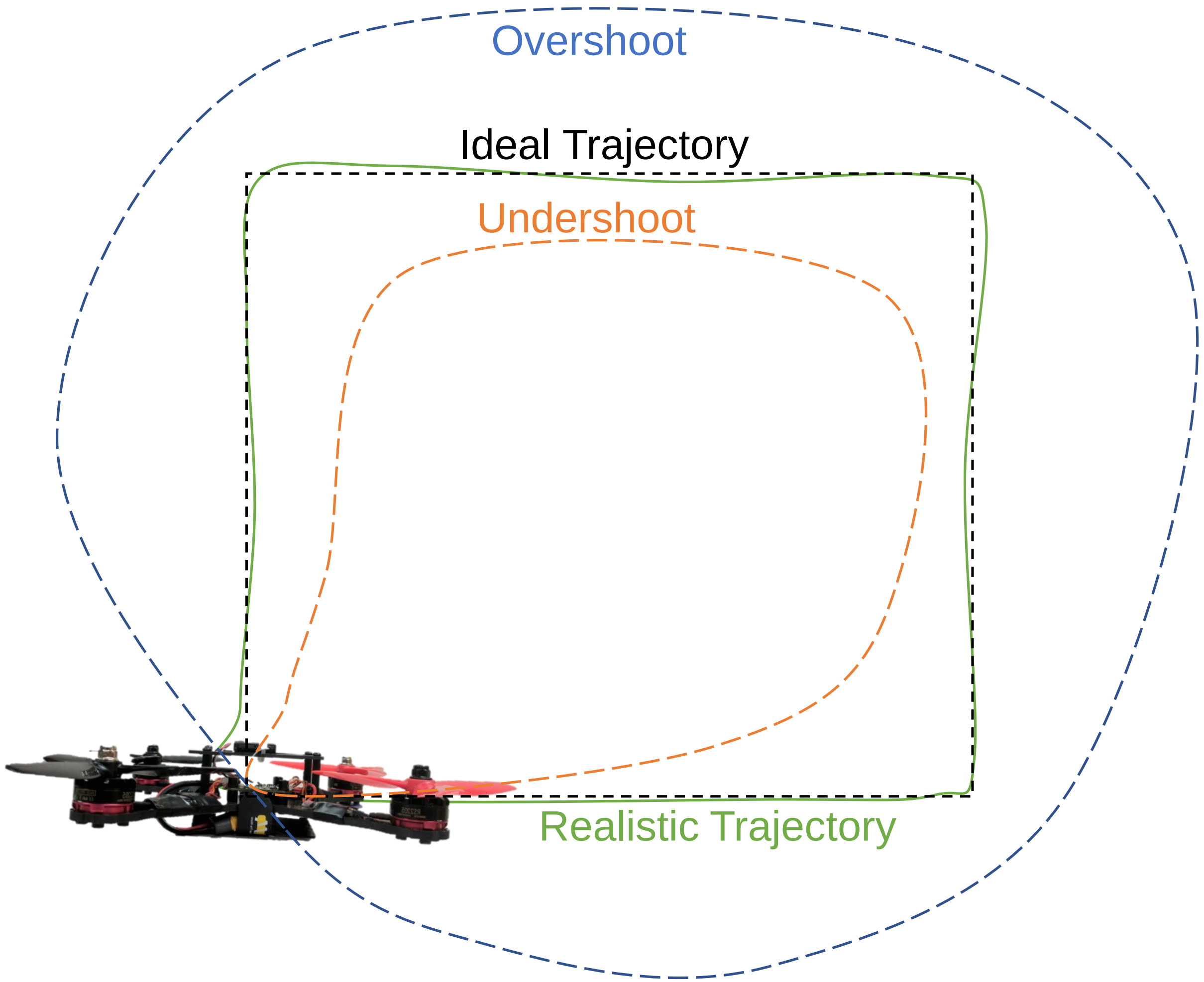
3D Trajectory Controller with ‘Simple’ Error Metric

Near-hover assumptions hold

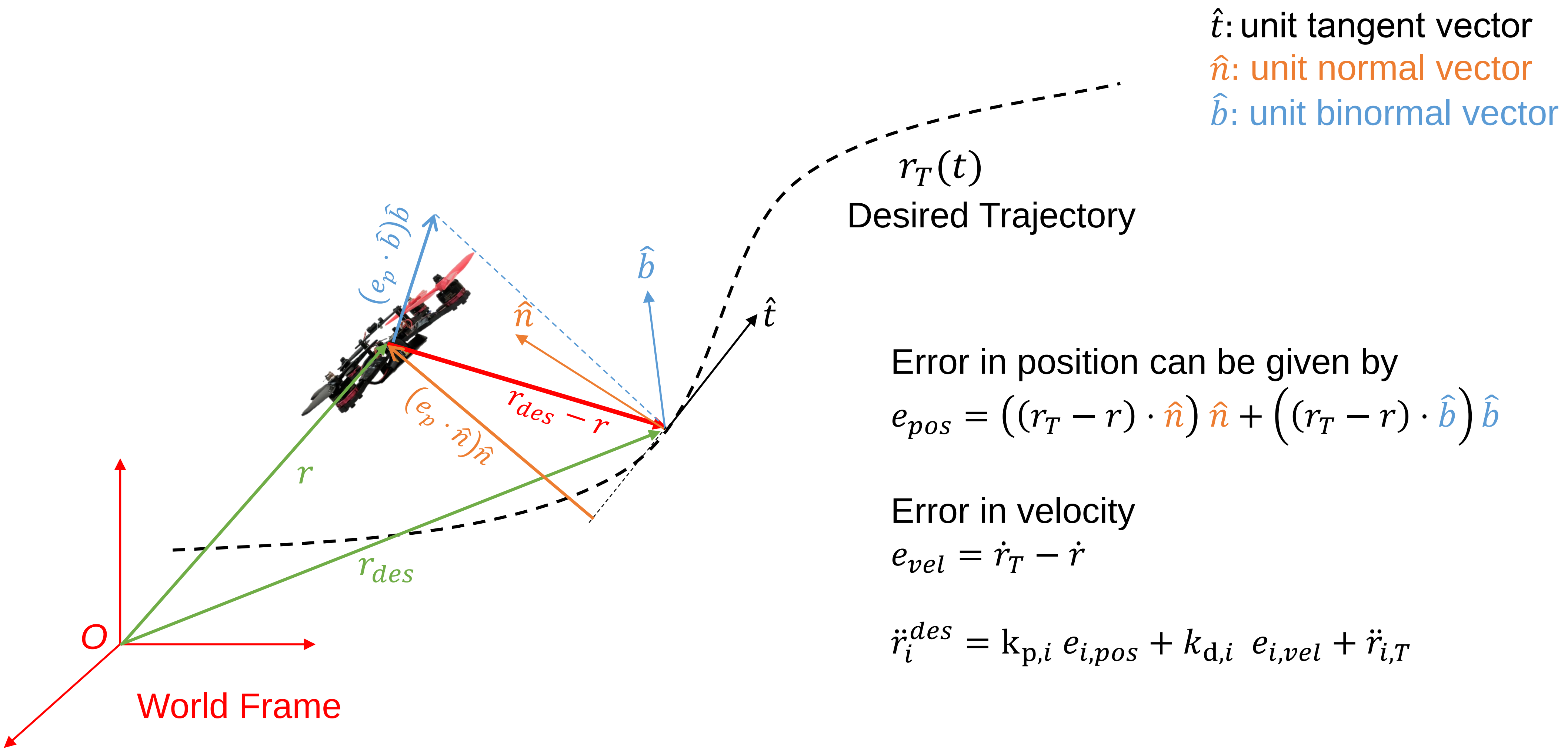




Problems with 'Simple' Error Metric



3D Trajectory Controller



Error in position can be given by

$$e_{pos} = \left((r_T - r)\cdot\hat{n}\right)\hat{n} + \left((r_T - r)\cdot\hat{b}\right)\hat{b}$$

Error in velocity

$$e_{vel} = \dot{r}_T - \dot{r}$$

$$\ddot{r}_i^{des} = \mathrm{k}_{\mathrm{p},i}\, e_{i,pos} + k_{\mathrm{d},i}\, e_{i,vel} + \ddot{r}_{i,T}$$